Name: Key Date: Band:
Algebra 2

Rational Exponents and Radical Functions Practice Problems

6.1 nth Roots and Rational Exponents

1. Evaluate $8^{\frac{4}{3}}$ without using a calculator.

$8^{\frac{4}{3}} = \sqrt[3]{8^4} = 2^4 = \boxed{16}$

2. Find the real solution(s) of $x^4 - 45 = 580$.

$+45 \quad +45$

$x^4 = 625$

$\sqrt[4]{x^4} = \sqrt[4]{625}$

$\boxed{x = 5}$

Evaluate the expression without using a calculator.

3. $8^{\frac{7}{3}}$

$8^{\frac{7}{3}} = \sqrt[3]{8^7} = 2^7 = \boxed{128}$

4. $9^{\frac{5}{2}}$

$9^{\frac{5}{2}} = \sqrt{9^5} = 3^5 = \boxed{243}$

5. $(-27)^{-\frac{2}{3}}$

$(-27)^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{(-27)^2}} = \frac{1}{(-3)^2} = \boxed{\frac{1}{9}}$

Find the real solution(s) of the equation. Round your answer to two decimal places when appropriate.

6. $x^5 + 17 = 35$

$-17 \quad -17$

$x^5 = 18$

$\sqrt[5]{x^5} = \sqrt[5]{18}$

$\boxed{x \approx 1.78}$

7. $\frac{7x^3}{7} = \frac{189}{7}$

$x^3 = 27$

$\sqrt[3]{x^3} = \sqrt[3]{27}$

$\boxed{x = 3}$

8. $(x+8)^4 = 16$

$\sqrt[4]{(x+4)^4} = \sqrt[4]{16}$

$x + 4 = 2$

$-4 \quad -4$

$\boxed{x = -2}$

6.2 Properties of Rational Exponents and Radicals

9. Use the properties of rational exponents to simplify $\left(\frac{54^{\frac{1}{3}}}{2^{\frac{1}{3}}}\right)^4$.

$\left(\frac{54^{1/3}}{2^{1/3}}\right)^4 = \left(27^{1/3}\right)^4 = 3^4 = \boxed{81}$

10. Write $\sqrt[4]{16x^{13}y^8z^7}$ in simplest form.

$2 \cdot \sqrt[4]{x^{12}} \cdot \sqrt[4]{x} \cdot y^2 \cdot \sqrt[4]{z^4} \cdot \sqrt[4]{z^3}$

$2x^3y^2z\sqrt[4]{xz^3}$

Simplify the expression.

11. $\left(\frac{6^{\frac{1}{5}}}{6^{\frac{2}{5}}}\right)^3$

$(6^{-\frac{1}{5}})^3 = 6^{-\frac{3}{5}} = \frac{1}{6^{3/5}}$

12. $\sqrt[4]{32} \cdot \sqrt[4]{8}$

$\sqrt[4]{256}$

4

13. $\frac{1}{2-\sqrt[4]{9}} \cdot \frac{2+\sqrt[4]{9}}{2+\sqrt[4]{9}}$

$\frac{2+\sqrt[4]{9}}{4-\sqrt{9}} \cdot \frac{4+\sqrt{9}}{4+\sqrt{9}} = \frac{\quad}{16-9}$

$\frac{8+2\sqrt{9}+4\sqrt[4]{9}+\sqrt[4]{9^3}}{7}$

14. $4\sqrt[5]{8} + 3\sqrt[5]{8}$

$7\sqrt[5]{8}$

15. $2\sqrt{48} - \sqrt{3}$

$2\sqrt{16}\sqrt{3} - \sqrt{3}$

$2 \cdot 4\sqrt{3} - \sqrt{3}$

$8\sqrt{3} - \sqrt{3}$

$7\sqrt{3}$

16. $\left(5^{\frac{2}{3}} \cdot 2^{\frac{3}{2}}\right)^{\frac{1}{2}}$

$5^{\frac{2}{6}} \cdot 2^{\frac{3}{4}}$

$5^{1/3} \cdot 2^{3/4}$

Simplify the expression. Assume all variables are positive.

17. $\sqrt[3]{125z^9}$

$5z^3$

18. $\frac{2^{\frac{1}{4}}z^{\frac{5}{4}}}{6z}$

$\frac{2^{1/4}z^{1/4}}{6}$

19. $\sqrt{10z^5} - z^2\sqrt{40z}$

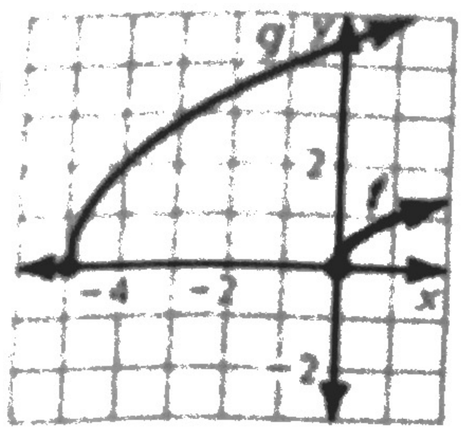
$\sqrt{z^2}\sqrt{10z} - z^2\sqrt{4}\sqrt{10z}$

$z^2\sqrt{10z} - 2z^2\sqrt{10z}$

$-z^2\sqrt{10z}$

6.3 Graphing Radical Functions

20. Describe the transformation of $f(x) = \sqrt{x}$ represented by $g(x) = 2\sqrt{x+5}$. Then graph each function.



vertical stretch by a factor of 2

horizontal translation left 5 units

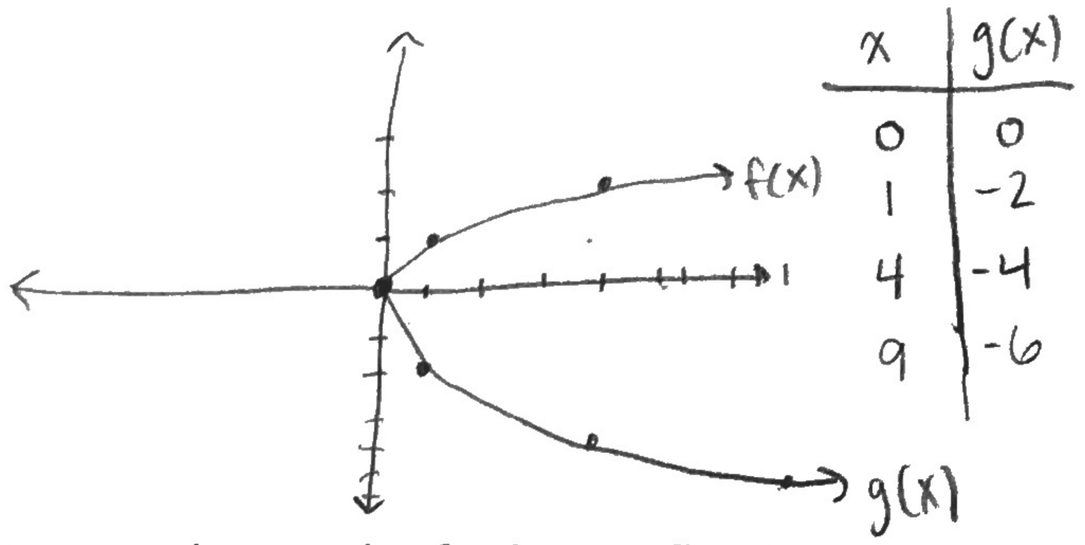
whoops!

Describe the transformation of f represented by g. Then graph each function.

21. $f(x) = \sqrt{x}, g(x) = -2\sqrt{x}$

reflection over x-axis

vertical stretch by a factor of 2

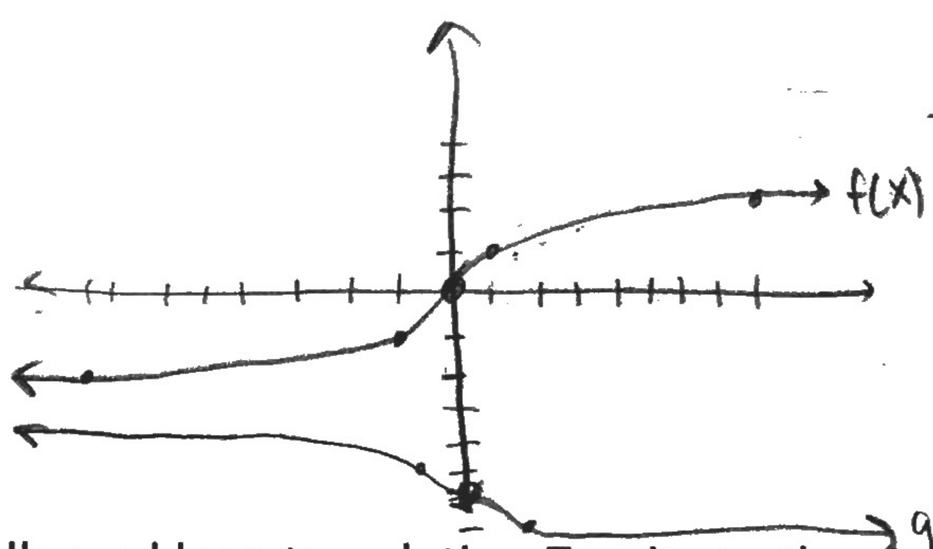


x	g(x)
0	0
1	-2
4	-4
9	-6

22. $f(x) = \sqrt[3]{x}, g(x) = \sqrt[3]{-x} - 6$

reflection over y-axis

vertical translation down 6 units



x	g(x)
-2	-4.7
-1	-5
0	-6
1	-7
2	-7.3

23. Let the graph of g be a reflection in the y-axis, followed by a translation 7 units to the right of the graph of $f(x) = \sqrt[3]{x}$. Write a rule for g.

① reflection in y-axis

$h(x) = f(-x)$

$h(x) = \sqrt[3]{-x}$

② translation 7 units right

$g(x) = h(x-7)$

$\boxed{g(x) = \sqrt[3]{-(x-7)}}$

6.4 Solving Radical Equations and Inequalities

24. Solve $\frac{6\sqrt{x+2}}{6} < \frac{18}{6}$.

$\sqrt{x+2} < 3$

$(\sqrt{x+2})^2 < 3^2$

$x+2 < 9$

$x+2 < 9$

$-2 \quad -2$

$\boxed{x < 7}$

Solve the equation. Check your solution.

25. $\frac{4\sqrt[3]{2x+1}}{4} = \frac{20}{4}$

$\sqrt[3]{2x+1} = 5$

$(\sqrt[3]{2x+1})^3 = 5^3$

$2x+1 = 125$

$2x = 124$

$\boxed{x = 62}$

26. $\sqrt{4x-4} = \sqrt{5x-1} - 1$

$(\sqrt{4x-4})^2 = (\sqrt{5x-1} - 1)^2$

$4x-4 = 5x-1-2\sqrt{5x-1}+1$

$-x-4 = -2\sqrt{5x-1}$

$(0.5x+2)^2 = (\sqrt{5x-1})^2$

$0.25x^2+2x+4 = 5x-1$

$4[0.25x^2-3x+5=0]$

$x^2-12x+20=0$

$(x-10)(x-2)=0$

$x-10=0$

$\boxed{x=10}$

$x-2=0$

$\boxed{x=2}$

27. $(6x)^{\frac{2}{3}} = 36$

$[(6x)^{2/3}]^{3/2} = 36^{3/2}$

$6x = 216$

$\boxed{x = 36}$

Solve the inequality.

28. $5\sqrt{x} + 2 > 17$

$5\sqrt{x} > 15$

$\sqrt{x} > 3$

$\boxed{x > 9}$

29. $2\sqrt{x-8} < 24$

$\sqrt{x-8} < 12$

$x-8 < 144$

$\boxed{x < 152}$

30. $7\sqrt[3]{x-3} \geq 21$

$3\sqrt{x-3} \geq 3$

$x-3 \geq 27$

$\boxed{x \geq 30}$

31. In a tsunami, the wave speeds (in meters per second) can be modeled by $s(d) = \sqrt{9.8d}$, where d is the depth (in meters) of the water. Estimate the depth of the water when the wave speed is 200 meters per second.

$200 = \sqrt{9.8d}$

$40000 = 9.8d$

$\boxed{4081\text{ m} \approx d}$