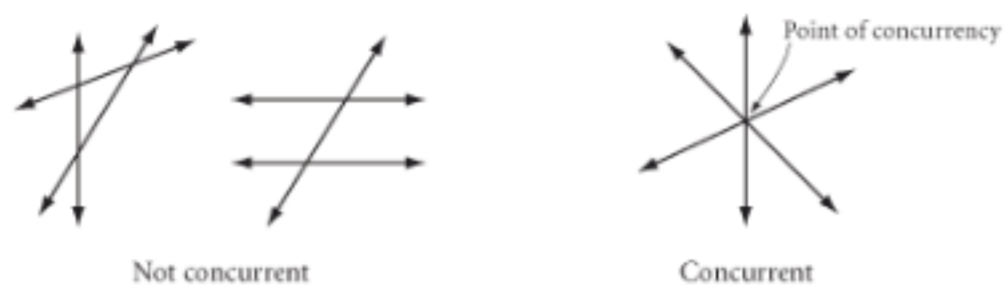
Name:_______________________________ Date:___________________________ Band:_____
Geometry

Points of Concurrency Activity

Instructions: You NEED a compass and straightedge to complete the activity. Work as a group for each construction.

You now can perform a number of constructions in triangles, including angle bisectors and perpendicular bisectors. You will also learn how to construct medians and altitudes. In this activity, you will discover special properties of these lines and segments. When three or more lines have a point in common, they are **concurrent**. Segments, rays, and even planes are concurrent if they intersect at a single point.



The point of intersection is the **point of concurrency.**

Construction 1: Circumcenter

Step 1: Draw a large triangle below. Make sure you have at least one acute triangle, one obtuse triangle, and one right triangle in your group.

Step 2: Construct the perpendicular bisector for each side of the triangle. The point of concurrency for the three perpendicular bisectors is the **circumcenter**.

Step 3: Measure and compare the distances from the circumcenter to each of the three vertices of the triangle. Are they the same?

Compare the distances from the circumcenter to each of the three sides of the triangle. Are they the same?

Step 4: Use a compass to construct a circle with the circumcenter as the center and that passes through all of the triangle's vertices. What do you notice?

Complete the theorem.

Circumcenter Theorem: The circumcenter of a triangle is ______________________________.

Construction 2: Incenter

Step 1: Draw a large triangle below. Make sure you have at least one acute triangle, one obtuse triangle, and one right triangle in your group.

Step 2: Construct the angle bisectors for each angle of the triangle. The point of concurrency for the three angle bisectors is the **incenter**.

Step 3: Measure and compare the distances from the incenter to each of the triangle's three sides. (Remember to use the perpendicular distance.) Are they the same?

Step 4: Construct the perpendicular segment from the incenter to any one of the three sides of the triangle. Mark the intersection between the perpendicular segment and the side of the triangle.

Step 5: Use a compass to construct a circle with the incenter as the center and that passes through the point of intersection marked in Step 4. What do you notice?

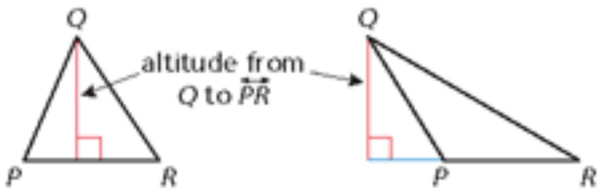
Complete the theorem.

Incenter Theorem: The incenter of a triangle is ____.

<u>Construction 3: Orthocenter</u>

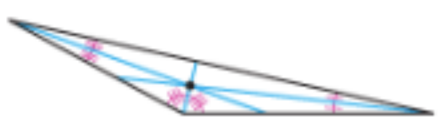
Step 1: Draw a large triangle. Make sure you have at least one acute triangle, one obtuse triangle, and one right triangle in your group.

Step 2: Construct the three **altitudes** of your triangle by drawing a perpendicular segment from a triangle's vertex to the opposite side. The point of concurrency for the three altitudes is called the **orthocenter.**



***For what kind of triangle will the points of concurrency be the same point?**

Recap:



Three angle bisectors
(incenter)

Three perpendicular bisectors
(circumcenter)

Three altitudes
(orthocenter)

Three medians
?

<u>Construction 4: Centroid</u>

Each person in your group should draw a different triangle for this construction. Make sure you have at least one acute triangle, one obtuse triangle, and one right triangle in your group.

Step 1: Draw a large scalene triangle and label it CNR. Locate the midpoints of the three sides. Construct the three **medians** by drawing a line segment from a triangle's vertex to the midpoint of the opposite side. (see "Three medians" in above recap diagram.) The point of concurrency of the three medians is the **centroid.**

Step 2: Label the three medians $\overline{CT}$, $\overline{NO}$, and $\overline{RE}$. Label the centroid D.

Step 3: Use your compass or ruler to investigate whether there is anything special about the centroid. Is the centroid equidistant from the three vertices?

From the three sides?

Is the centroid the midpoint of each median?

Step 4: The centroid divides the median into two segments, a long segment and a short segment. Focus on one median. Use your compass or ruler to compare the length of the longer segment to the length of the shorter segment and find the ratio.

Step 5: Find the ratios of the lengths of the segment parts for the other two medians. Do you get the same ratio for each median?

Complete the theorem.

Centroid Theorem: The centroid of a triangle is ____.