Name: key Date: Band:
Algebra 2

Unit 5 Polynomial Functions Study Guide

4.1 Adding, Subtracting, and Multiplying Polynomials

1. Multiply $(x-2)(x-1)(x+3)$

$(x-2)(x^2+3x-x-3)$

$(x-2)(x^2+2x-3)$

$x^3+2x^2-3x-2x^2-4x+6$

$\boxed{x^3-7x+6}$

2. Expand $(4x+2)^4 = (4x+2)(4x+2)(4x+2)(4x+2)$

$(16x^2+8x+8x+4)(16x^2+8x+8x+4)$

$(16x^2+16x+4)(16x^2+16x+4)$

$256x^4+256x^3+64x^2+256x^3+256x^2+64x+64x^2+64x+16$

$\boxed{256x^4+512x^3+384x^2+128x+16}$

Find the sum or difference.

probably should have been cubed oops!

3. $(4x^2-12x^2-5)-(-8x^2+4x+3)$

$4x^2-12x^2-5+8x^2-4x-3$

$\boxed{-4x-8}$

4. $(x^4+3x^3-x^2+6)+(2x^4-3x+9)$

$\boxed{3x^4+3x^3-x^2-3x+15}$

5. $(3x^2+9x+13)-(x^2-2x+12)$

$3x^2+9x+13-x^2+2x-12$

$\boxed{2x^2+11x+1}$

Find the product.

6. $(2y^2+4y-7)(y+3)$

$2y^3+6y^2+4y^2+12y-7y-21$

$\boxed{2y^3+10y^2+5y-21}$

7. $(2m+n)^3$

$(2m+n)(2m+n)(2m+n)$

$(2m+n)(4m^2+2mn+2mn+n^2)$

$(2m+n)(4m^2+4mn+n^2)$

$8m^3+8m^2n+2mn^2+4m^2n+4mn^2+n^3$

$\boxed{8m^3+12m^2n+6mn^2+n^3}$

8. $(s+2)(s+4)(s-3)$

$(s+3)(s^2-3s+4s-12)$

$(s+3)(s^2+s-12)$

$s^3+s^2-12s+3s^2+3s-36$

$\boxed{s^3+4s^2-9s-36}$

3 1 3 3 1
4 1 4 6 4 1
5 1 5 10 10 5 1
6 1 6 15 20 15 6 1

Expand the binomial.

9. $(m + 4)^4$ **10.** $(3s + 2)^5$ **11.** $(z + 1)^6$

$m^4 + 4m^3(4) + 6m^2(4)^2 + 4m(4)^3 + (4)^4$

$\boxed{m^4 + 16m^3 + 96m^2 + 256m + 256}$

$(3s)^5 + 5(3s)^4(2) + 10(3s)^3(2)^2 + 10(3s)^2(2)^3 + 5(3s)(2)^4 + 2^5$

$\boxed{243s^5 + 810s^4 + 1080s^3 + 720s^2 + 240s + 32}$

$z^6 + 6z^5(1) + 15z^4(1)^2 + 20z^3(1)^3 + 15z^2(1)^4 + 6z(1)^5 + 1^6$

$\boxed{z^6 + 6z^5 + 15z^4 + 20z^3 + 15z^2 + 6z + 1}$

4.2 Dividing Polynomials

12. Use synthetic division to evaluate $f(x) = -2x^3 + 42x^2 + 8x + 10$ when $x = -3$. Remainder Theorem

-3	-2	42	8	10
		6	-144	408
	-2	48	-136	418

$\boxed{f(-3) = 418}$

Divide using polynomial long division or synthetic division.

13. $(x^3 + x^2 + 3x - 4) \div (x^2 + 2x + 1)$ needs long division

$$\begin{array}{r} x - 1 \\ x^2 + 2x + 1 \overline{) x^3 + x^2 + 3x - 4} \\ -(x^3 + 2x^2 + x) \\ -x^2 + 2x - 4 \\ -(-x^2 - 2x - 1) \\ 4x - 3 \end{array}$$

$\boxed{x - 1 + \frac{4x-3}{x^2+2x+1}}$

14. $(x^4 + 3x^3 - 4x^2 + 5x + 3) \div (x^2 + x + 4)$ needs long division

$$\begin{array}{r} x^2 + 2x - 10 \\ x^2 + x + 4 \overline{) x^4 + 3x^3 - 4x^2 + 5x + 3} \\ -(x^4 + x^3 + 4x^2) \\ 2x^3 - 8x^2 + 5x + 3 \\ -(2x^3 + 2x^2 + 8x) \\ -10x^2 - 3x + 3 \\ -(-10x^2 - 10x - 40) \\ 7x + 43 \end{array}$$

$\boxed{x^2 + 2x - 10 + \frac{7x+43}{x^2+x+4}}$

15. $(x^4 - x^2 - 7) \div (x+4)$ can use synthetic division because divisor is a binomial

$$\begin{array}{r|rrrrr} -4 & 1 & 0 & -1 & 0 & -7 \\ & & -4 & 16 & -60 & 240 \\ \hline & 1 & -4 & 15 & -60 & 233 \\ & x^3 & x^2 & x & \# & \text{remander} \end{array}$$

$$\boxed{x^3 - 4x^2 + 15x - 60 + \frac{233}{x+4}}$$

16. Use synthetic division to evaluate $g(x) = 4x^3 + 2x^2 - 4$ when $x = 5$. Remainder theorem

$$\begin{array}{r|rrrr} 5 & 4 & 2 & 0 & -4 \\ & & 20 & 110 & 550 \\ \hline & 4 & 22 & 110 & 546 \end{array}$$

$\boxed{g(5) = 546}$

4.3 Factoring Polynomials

17. Factor $x^4 + 8x$ completely.

$x(x^3+8)$

$\boxed{x(x+2)(x^2-2x+4)}$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

18. Determine whether $x + 4$ is a factor of $f(x) = x^5 + 4x^4 + 2x + 8$. The factor theorem

$$\begin{array}{r|rrrrr} -4 & 1 & 4 & 0 & 2 & 8 \\ & & -4 & 0 & 0 & -8 \\ \hline & 1 & 0 & 0 & 2 & 0 \end{array}$$

remander = 0

so $x+4$ is a factor

of $f(x)$

Factor the polynomial completely.

19. $64x^3 - 8$

$8(8x^3-1)$

$\boxed{8(2x-1)(4x^2+2x+1)}$

20. $2z^5 - 12z^3 + 10z$

$2z(z^4-6z^2+5)$

$2z(z^2-1)(z^2-5)$

$\boxed{2z(z-1)(z+1)(z^2-5)}$

21. $2a^3 - 7a^2 - 8a + 28$

$a^2(2a-7)-4(2a-7)$

$(a^2-4)(2a-7)$

$\boxed{(a-2)(a+2)(2a-7)}$

22. Show that $x + 2$ is a factor of $f(x) = x^4 + 2x^3 - 27x - 54$. Then factor $f(x)$ completely. factor Theorem

$$\begin{array}{r|rrrrr} -2 & 1 & 2 & 0 & -27 & -54 \\ & & -2 & 0 & 0 & 54 \\ \hline & 1 & 0 & 0 & -27 & 0 \\ & x^3 & x^2 & x & \# & \end{array}$$

$r=0$ so $x+2$ is a factor

x^3-27 ← other factor

$f(x)=x^4+2x^3-27x-54$

$f(x)=(x+2)(x^3-27)$

$\boxed{f(x)=(x+2)(x-3)(x^2+3x+9)}$

4.4 Solving Polynomial Equations

23. Find all real solutions of $x^3 + x^2 - 8x - 12 = 0$. cannot be factored

* can also graph and calc x-ints.

Algebraic method (Rational Root Theorem)

LC = 1 CT = −12 (factors = 1, 2, 3, 4, 6, 12)

possible rational solutions = $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}$

Test $x=1$ (not r=0)

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -8 & -12 \\ & & 1 & 2 & -6 \\ \hline & 1 & 2 & -6 & -18 \neq 0 \end{array}$$

Test $x=-2$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -8 & -12 \\ & & -2 & 2 & 12 \\ \hline & 1 & -1 & -6 & 0 \\ & x^2 & x & \# & \end{array}$$

0 r=0 yay!

$x^3+x^2-8x-12=0$

$(x-2)(x^2-x-6)=0$

$(x-2)(x-3)(x+2)=0$

$x-2=0 \quad x-3=0 \quad x+2=0$

$\boxed{x=2} \quad \boxed{x=3} \quad \boxed{x=-2}$

24. Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the zeros -4 and $1+\sqrt{2}$. ↳ no repeated/imaginary solutions

a=1

zeros = $-4, 1+\sqrt{2}, 1-\sqrt{2}$ (Irrational Conjugates theorem)

$f(x)=(x+4)(x-(1+\sqrt{2}))(x-(1-\sqrt{2}))$

$f(x)=(x+4)([x-1]-\sqrt{2})([x-1]+\sqrt{2})$

$f(x)=(x+4)((x-1)^2-2)$

$f(x)=(x+4)(x^2-2x+1+2)$

$f(x)=(x+4)(x^2-2x+3)$

$f(x)=x^3-2x^2+3x+4x^2-8x+12$

$\boxed{f(x)=x^3+2x^2-5x+12}$

Rational Root Theorem
↑
Find all real solutions of the equation. ★ could graph and calc x-ints.

25. $x^3 + 3x^2 - 10x - 24 = 0$ cannot be factored

$LC = 1$ $\quad CT = -24$ (factors $= 1, 2, 3, 4, 6, 8, 12, 24$)

possible rational solutions $= \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{8}{1}, \pm\frac{12}{1}, \pm\frac{24}{1}$

Test $x = -2$

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -10 & -24 \\ & & -2 & -2 & 24 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$x^2 + x - 12$ ← other factor

$(x+2)(x^2+x-12) = 0$

$(x+2)(x+4)(x-3) = 0$

$x+2=0 \quad x+4=0$

$\boxed{x=-2}$ $\boxed{x=-4}$

$x-3=0$

$\boxed{x=3}$

26. $x^3 + 5x^2 - 2x - 24 = 0$ cannot be factored

$LC = 1$ $\quad CT = -24$ (factors $= 1, 2, 3, 4, 6, 8, 12, 24$)

possible rational solutions $= \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{8}{1}, \pm\frac{12}{1}, \pm\frac{24}{1}$

Test $x = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 5 & -2 & -24 \\ & & 2 & 14 & 24 \\ \hline & 1 & 7 & 12 & 0 \end{array}$$

$x^2 + 7x + 12$ ← other factor

$(x-2)(x^2+7x+12) = 0$

$(x-2)(x+4)(x+3) = 0$

$x-2=0 \quad x+4=0 \quad x+3=0$

$\boxed{x=2}$ $\boxed{x=-4}$ $\boxed{x=-3}$

Irrational Conjugates Theorem

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros. ↳ no repeated / imaginary solutions $\quad a=1$

27. 1, $2 - \sqrt{3}$, also $2+\sqrt{3}$

$f(x) = (x-1)(x-(2-\sqrt{3}))(x-(2+\sqrt{3}))$

$f(x) = (x-1)((x-2)+\sqrt{3})((x-2)-\sqrt{3})$

$f(x) = (x-1)((x-2)^2-3)$

$f(x) = (x-1)(x^2-4x+4-3)$

$f(x) = (x-1)(x^2-4x+1)$

$f(x) = x^3-4x^2+x-x^2+4x-1$

$\boxed{f(x) = x^3-5x^2+5x-1}$

28. 2, 3, $\sqrt{5}$, $-\sqrt{5}$

$f(x) = (x-2)(x-3)(x-\sqrt{5})(x+\sqrt{5})$

$f(x) = (x^2-5x+6)(x^2-5)$

$f(x) = x^4-5x^3+6x^2-5x^2+25x-30$

$\boxed{f(x) = x^4-5x^3+x^2+25x-30}$

29. $-2, 5, 3+\sqrt{6}$, $3-\sqrt{6}$

$f(x) = (x+2)(x-5)(x-(3+\sqrt{6}))(x-(3-\sqrt{6}))$

$f(x) = (x^2-3x-10)((x-3)-\sqrt{6})((x-3)+\sqrt{6})$

$f(x) = (x^2-3x-10)((x-3)^2-6)$

$f(x) = (x^2-3x-10)(x^2-6x+9-6)$

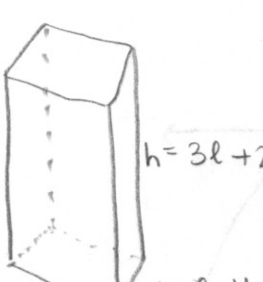
$f(x) = (x^2-3x-10)(x^2-6x+3)$

$f(x) = x^4-6x^3+3x^2-3x^3+18x^2-9x-10x^2+60x-30$

$\boxed{f(x) = x^4-9x^3+11x^2-51x-30}$

30. You use 240 cubic inches of clay to make a sculpture shaped as a rectangular prism. The width is 4 inches less than the length and the height is 2 inches more than three times the length. What are the dimensions of the sculpture? Justify your answer.

l = length



Volume = l × w × h

Volume = 240 in.^{3}

$240 = l(3l+2)(l-4)$

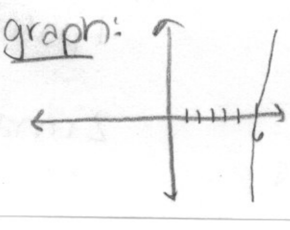
$240 = l(3l^2-12l+2l-8)$

$240 = l(3l^2-10l-8)$

$240 = 3l^3-10l^2-8l$

$0 = 3l^3-10l^2-8l-240$

graph:



$l = 6$ is a zero/solution/x-int.

dimensions: $l = \boxed{6 \text{ in}}$

$w = 6-4 = \boxed{2 \text{ in}}$

$h = 3(6)+2 = \boxed{20 \text{ in}}$

4.5 The Fundamental Theorem of Algebra

31. Find all zeros of $f(x) = x^4 + 2x^3 + 6x^2 + 18x - 27$. cannot factor but can graph and calc x-ints. but missing 2 imaginary zeros

Rational Root Theorem

LC = 1 CT = −27 (factors = 1, 3, 9, 27)

possible rational zeros $= \pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{9}{1}, \pm\frac{27}{1}$

Test x = 1

$$\begin{array}{c|ccccc} 1 & 1 & 2 & 6 & 18 & -27 \\ & & 1 & 3 & 9 & 27 \\ \hline & 1 & 3 & 9 & 27 & 0 \end{array}$$

$x^3+3x^2+9x+27$ other factor

$f(x) = (x-1)(x^3+3x^2+9x+27)$

$f(x) = (x-1)(x^2(x+3)+9(x+3))$

$f(x) = (x-1)(x^2+9)(x+3)$

$0 = (x+1)(x^2+9)(x+3)$

$0 = x+1$ $0 = x^2+9$ $0 = x+3$

$\boxed{x=-1}$ $-9 = x^2$ $\boxed{x = \pm 3i}$ $\boxed{x=-3}$

complex conjugates thrm **Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.** ↳ no repeated solutions a = 1

32. $3, 1+2i$, $1-2i$

$f(x) = (x-3)(x-(1+2i))(x-(1-2i))$

$f(x) = (x-3)((x-1)-2i)((x-1)+2i)$

$f(x) = (x-3)((x-1)^2-4i^2)$

$f(x) = (x-3)(x^2-2x+1+4)$

$f(x) = (x-3)(x^2-2x+5)$

$\boxed{f(x) = x^3-5x^2+11x-15}$

33. $-1, 2, 4i$, $-4i$

$f(x) = (x+1)(x-2)(x-4i)(x+4i)$

$f(x) = (x^2-x-2)(x^2-16i^2)$

$f(x) = (x^2-x-2)(x^2+16)$

$f(x) = x^4+16x^2-x^3-16x-2x^2-32$

$\boxed{f(x) = x^4-x^3+14x^2-16x-32}$

34. $-5, -4, 1-i\sqrt{3}$, $1+i\sqrt{3}$

$f(x) = (x+5)(x+4)(x-1)(x-(1-i\sqrt{3}))(x-(1+i\sqrt{3}))$

$f(x) = (x^2+9x+20)(x-1)((x-1)+i\sqrt{3})((x-1)-i\sqrt{3})$

$f(x) = (x^3+8x^2+11x-20)((x-1)^2-3i^2)$

$f(x) = (x^3+8x^2+11x-20)(x^2-2x+1+3)$

$f(x) = (x^3+8x^2+11x-20)(x^2-2x+4)$

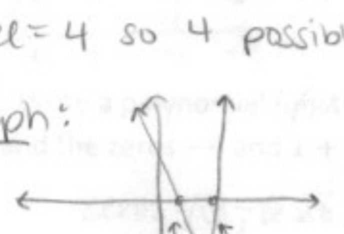
$\boxed{f(x) = x^5+6x^4-x^3+12x^2+62x-80}$

Determine the possible numbers of positive real zeros, negative real zeros, and imaginary zeros for the function.

35. $f(x) = x^4 - 10x + 8$

degree = 4 so 4 possible zeros

graph:



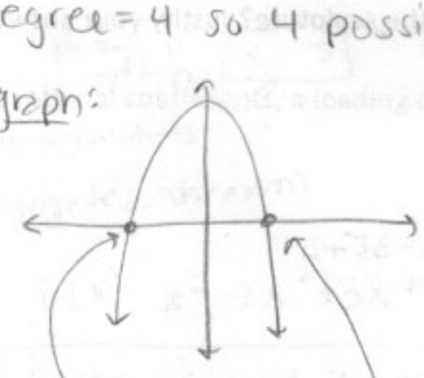
2 positive real zeros (x-ints.)

2 imaginary zeros (not shown on graph)

36. $f(x) = -6x^4 - x^3 + 3x^2 + 2x + 18$

degree = 4 so 4 possible zeros

graph:



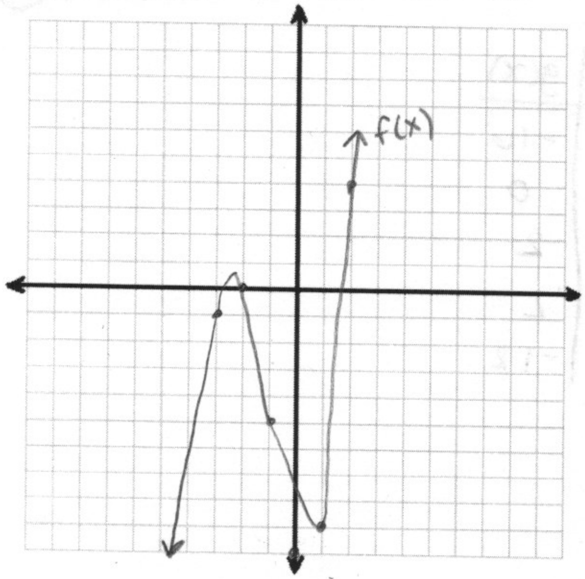
1 negative real zero (x-ints)

1 positive real zero

2 imaginary zeros (not shown on graph)

4.6 Graphing Polynomial Functions

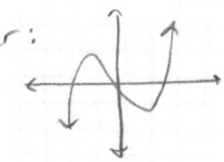
37. Graph $f(x) = x^3 + 3x^2 - 3x - 10$



LC = +

degree = odd

end behavior:



y-int = -10

x-ints = -2,

table:

x	$f(x)$
-3	-1
-2	0
-1	-5
0	-10
1	-9
2	4

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

38. $h(x) = -x^3 + 2x^2 - 15x^7$

polynomial function

$h(x) = -15x^7 - x^3 + 2x^2$

degree = 7 type = 7^{th} degree

LC = -15

39. $p(x) = x^3 - 5x^{0.5} + 13x^2 + 8$

not a polynomial function

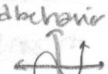
Graph the polynomial function.

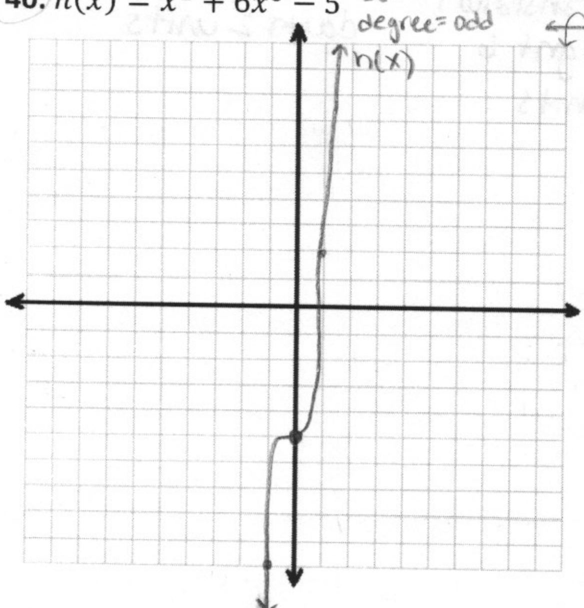
40. $h(x) = x^2 + 6x^5 - 5$

LC = +

degree = odd

end behavior:





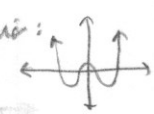
x	-1	0	1
$h(x)$	-10	-5	2

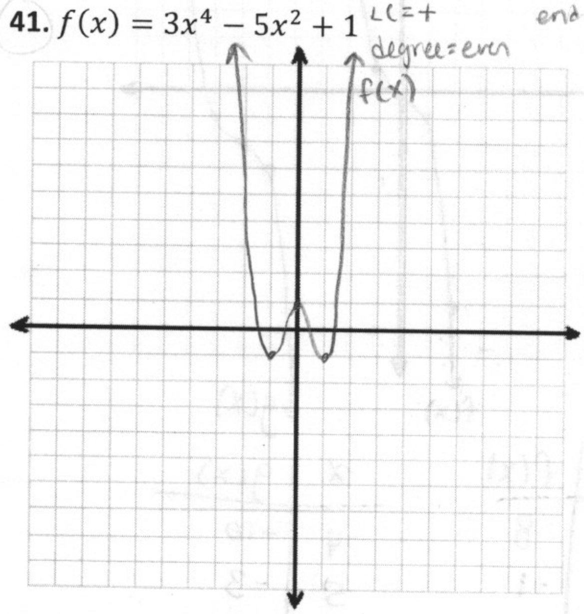
41. $f(x) = 3x^4 - 5x^2 + 1$

LC = +

degree = even

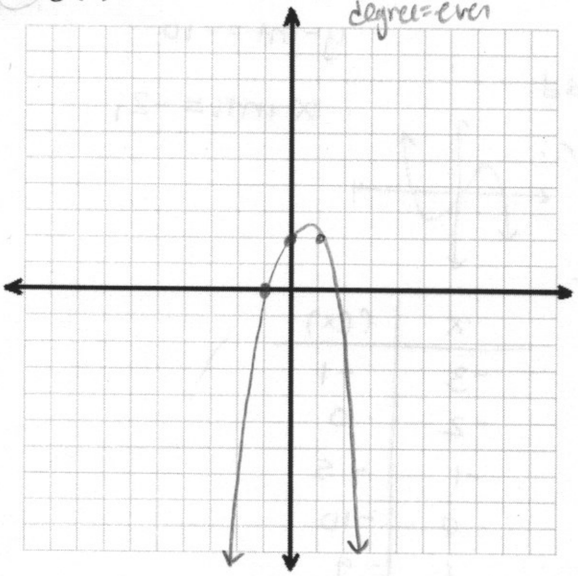
end behavior:





x	-2	-1	0	-1	2
$f(x)$	29	-1	1	1	29

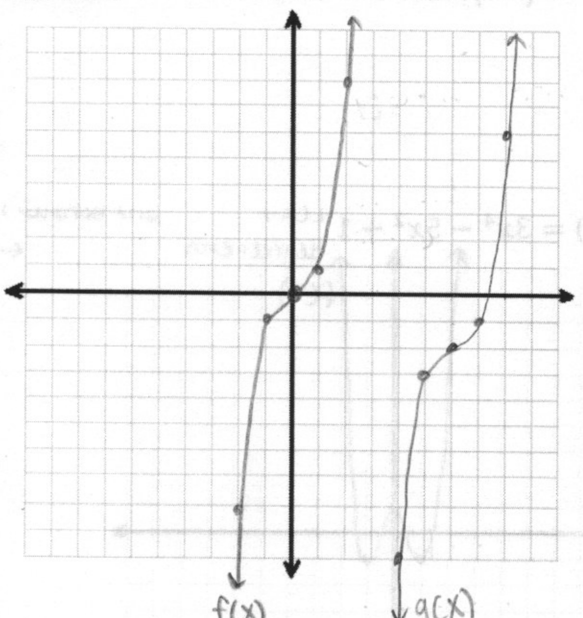
42. $g(x) = -x^4 + x + 2$ LC = − degree = even end behavior:



x	$g(x)$
-2	-16
-1	0
0	2
1	2
2	-12

4.7 Transformations of Polynomial Functions

43. Describe the transformation of $f(x) = x^3$ represented by $g(x) = (x-6)^3 - 2$. Then graph each function.



$g(x) = (x-6)^3 - 2$

horizontal translation right 6 units

vertical translation down 2 units

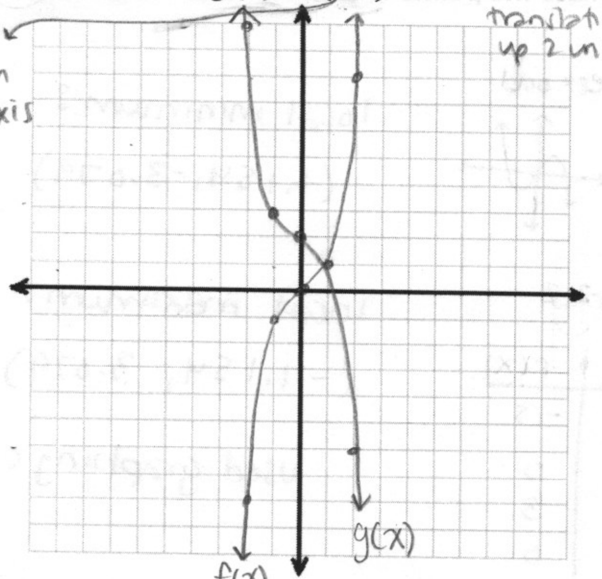
x	$f(x)$
-2	-8
-1	-1
0	0
1	1
2	8

x	$g(x)$
4	-10
5	-3
6	-2
7	-1
8	6

Describe the transformation of f represented by g. Then graph each function.

44. $f(x) = x^3, g(x) = (-x)^3 + 2$ → vertical translation up 2 units

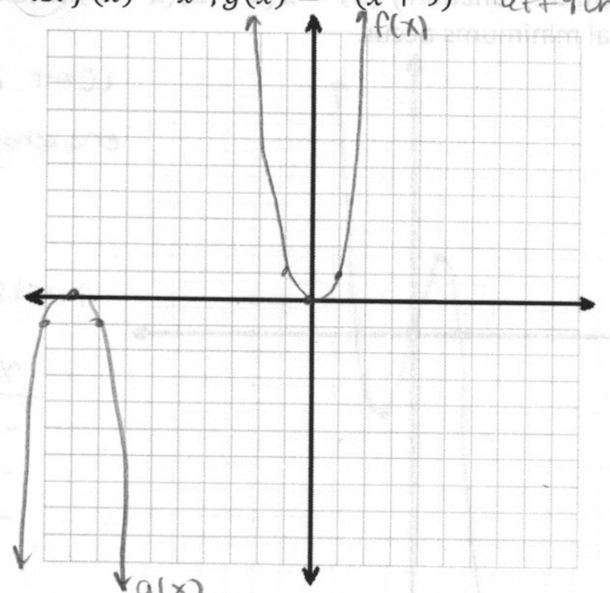
reflection over y-axis



45. $f(x) = x^4, g(x) = -(x+9)^4$

→ reflection over x-axis

horizontal translation left 9 units



Write a rule for g.

46. Let the graph of g be a horizontal shrink by a factor of 4, followed by a translation 3 units right and 5 units down of the graph of $f(x) = x^5 + 3x$.

① horizontal shrink by a factor of 4

$h(x) = f(4x)$

$h(x) = (4x)^5 + 3(4x)$

$h(x) = 1024x^5 + 12x$

② translation 3 units right and 5 units down

$g(x) = h(x-3) - 5$

$g(x) = (4(x-3))^5 + 3(4(x-3)) - 5$

$g(x) = (4x-12)^5 + 12x - 36 - 5$

$\boxed{g(x) = (4x-12)^5 + 12x - 41}$

47. Let the graph of g be a translation 5 units up, followed by a reflection in the y-axis of the graph of $f(x) = x^4 - 2x^3 - 12$.

① translation 5 units up

$h(x) = f(x) + 5$

$h(x) = x^4 - 2x^3 - 12 + 5$

$h(x) = x^4 - 2x^3 - 7$

② reflection in the y-axis

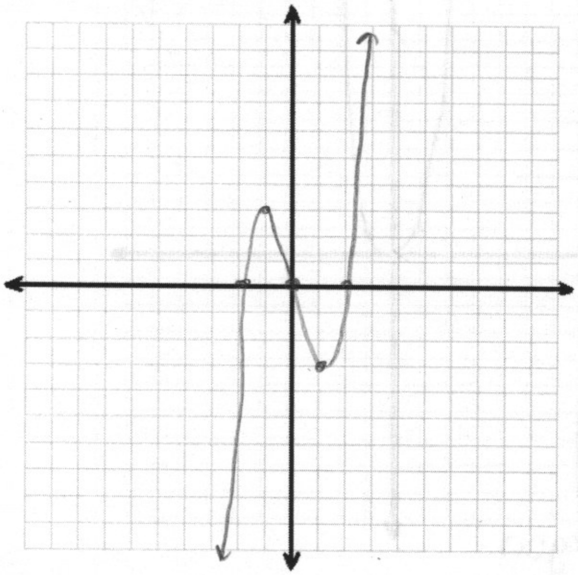
$g(x) = h(-x)$

$g(x) = (-x)^4 - 2(-x)^3 - 7$

$\boxed{g(x) = x^4 + 2x^3 - 7}$

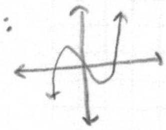
4.8 Analyzing Graphs of Polynomials Functions

48. Graph the function $f(x) = x(x+2)(x-2)$. Then estimate the points where the local maximums and local minimums occur.



LC = + degree = odd

end behavior:



local minimum:

(1.154, -3.079)

x-ints: 0, -2, 2

local maximum:

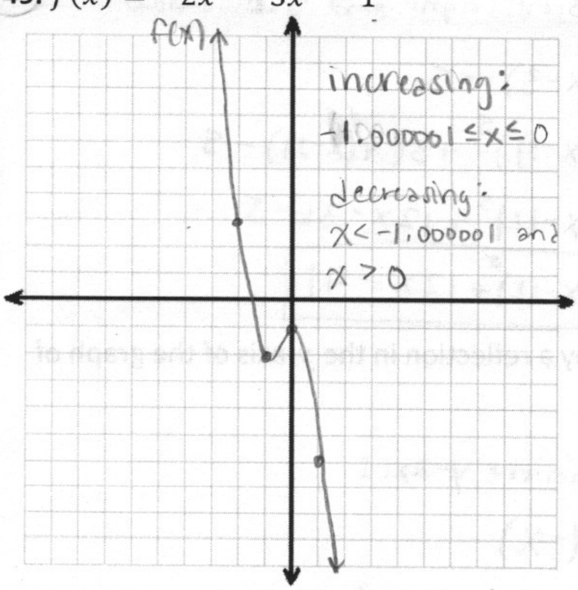
(-1.154, 3.079)

x	f(x)
-3	-15
-2	0
-1	3
0	0
1	-3
2	0
3	15

used graphing calc

Graph the function. Identify the x-intercepts and the points where the local maximums and local minimums occur. Determine the intervals for which the function is increasing or decreasing.

49. $f(x) = -2x^3 - 3x^2 - 1$



increasing:

-1.000001 ≤ x ≤ 0

decreasing:

x < -1.000001 and

x > 0

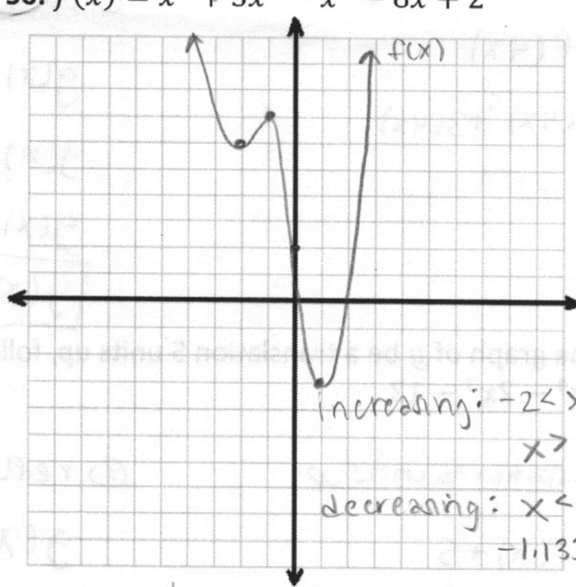
x	-3	-2	-1	0	1	2
f(x)	26	3	-2	-1	-6	-29

x-ints: (-1.677651, 0)

local max: (0, -1)

local min: (-1.000001, -2)

50. $f(x) = x^4 + 3x^3 - x^2 - 8x + 2$



increasing: -2 < x < -1.133

x > 0.883

decreasing: x < -2

-1.133 < x < 0.883

x	-3	-2	-1	0	1	2
f(x)	17	6	7	2	-3	22

x-ints: (.248, 0) and (1.345, 0)

local max: (-1.133, 7.065)

local mins: (-2, 6) and (0.883, -3.170)

even: $f(-x)=f(x)$ odd: $f(-x)=-f(x)$

Determine whether the function is *even, odd,* or *neither.*

51. $f(x) = 2x^3 + 3x$

$f(-x) = 2(-x)^3 + 3(-x)$

$f(-x) = -2x^3 - 3x = -f(x)$

$\boxed{\text{odd}}$

52. $g(x) = 3x^2 - 7$

$g(-x) = 3(-x)^2 - 7$

$g(-x) = 3x^2 - 7 = g(x)$

$\boxed{\text{even}}$

53. $h(x) = x^6 + 3x^5$

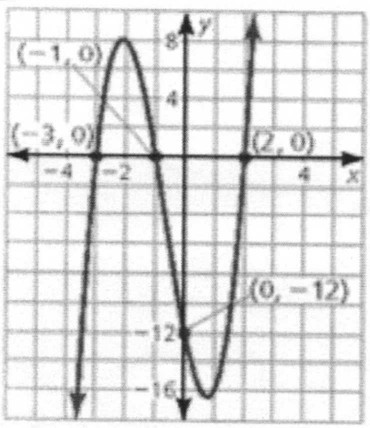
$h(-x) = (-x)^6 + 3(-x)^5$

$h(-x) = x^6 - 3x^5$

$\boxed{\text{neither}}$

4.9 Modeling with Polynomial Functions

54. Write the cubic function whose graph is shown.



$f(x) = a(x+3)(x+1)(x-2)$

$-12 = a(0+3)(0+1)(0-2)$

$-12 = a(3)(1)(-2)$

$-12 = -6a$

$2 = a$

$\boxed{f(x) = 2(x+3)(x+1)(x-2)}$

55. Write a cubic function whose graph passes through the points $(-4,0)$, $(4,0)$, $(0,6)$, and $(2,0)$.

y-int.

$f(x) = a(x+4)(x-4)(x-2)$

$6 = a(0+4)(0-4)(0-2)$

$6 = a(4)(-4)(-2)$

$6 = 32a$

$\frac{3}{16} = a$

$\boxed{f(x) = \frac{3}{16}(x+4)(x-4)(x-2)}$