Name: key Date: ______ Band: ______

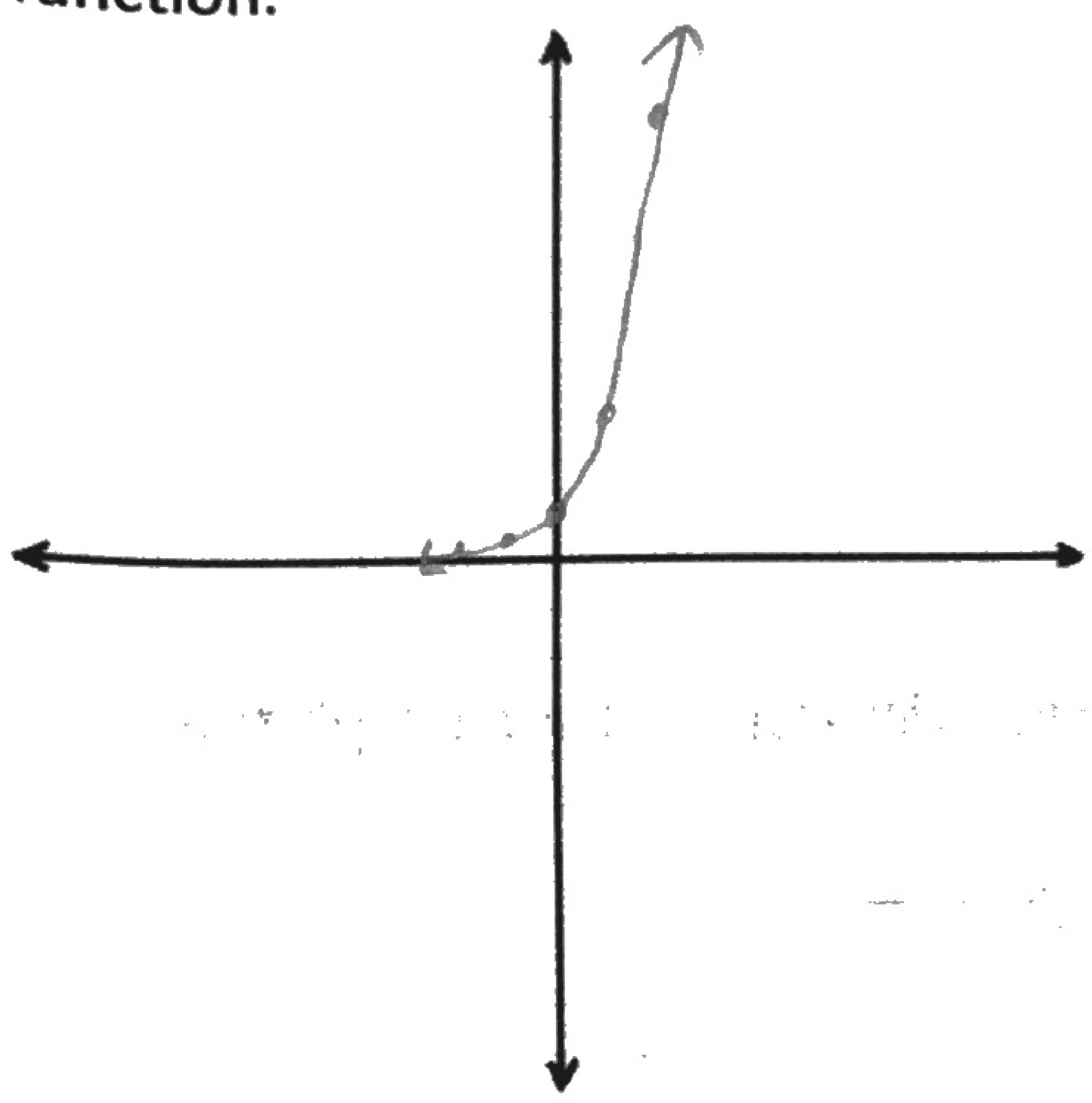
Algebra 2 Unit 7

Exponential and Logarithmic Functions Practice Problems

7.1 Exponential Growth and Decay

1. Tell whether the function $y = 3^x$ represents *exponential growth* or *exponential decay*. Then graph the function.

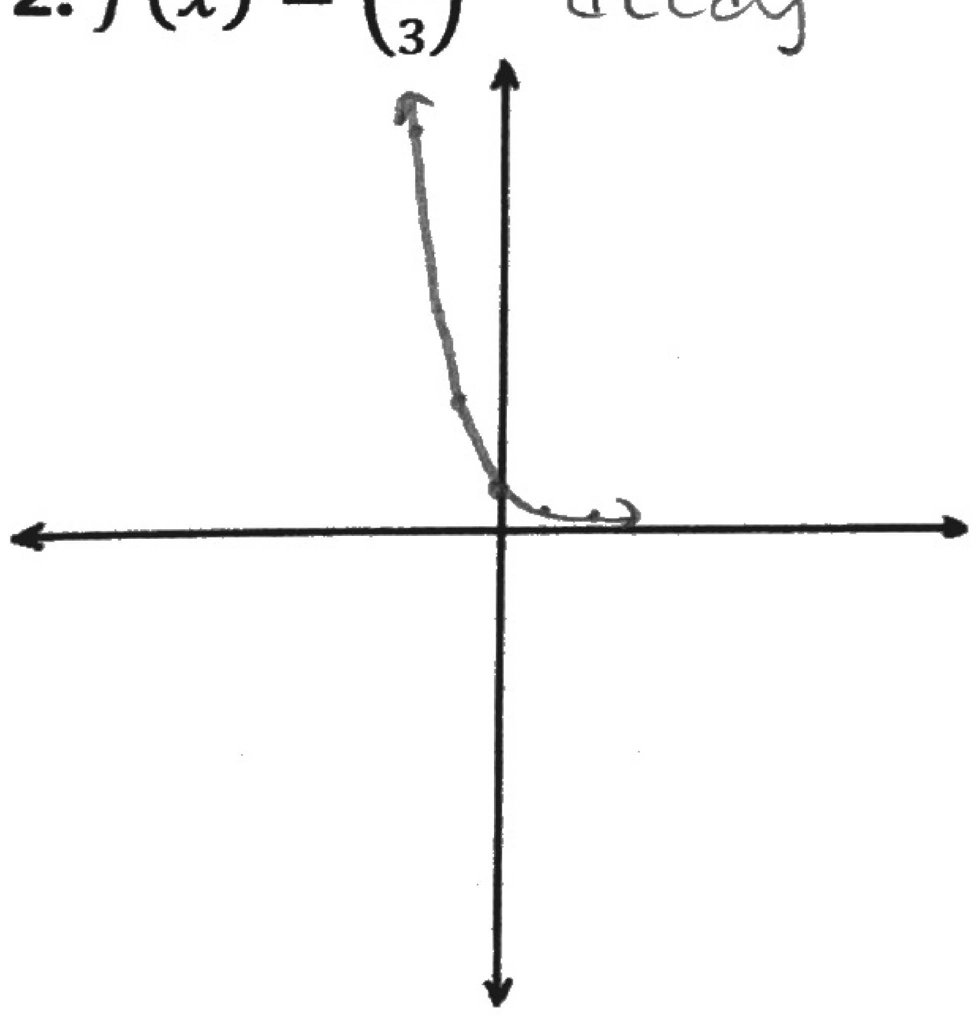
exponential growth



x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

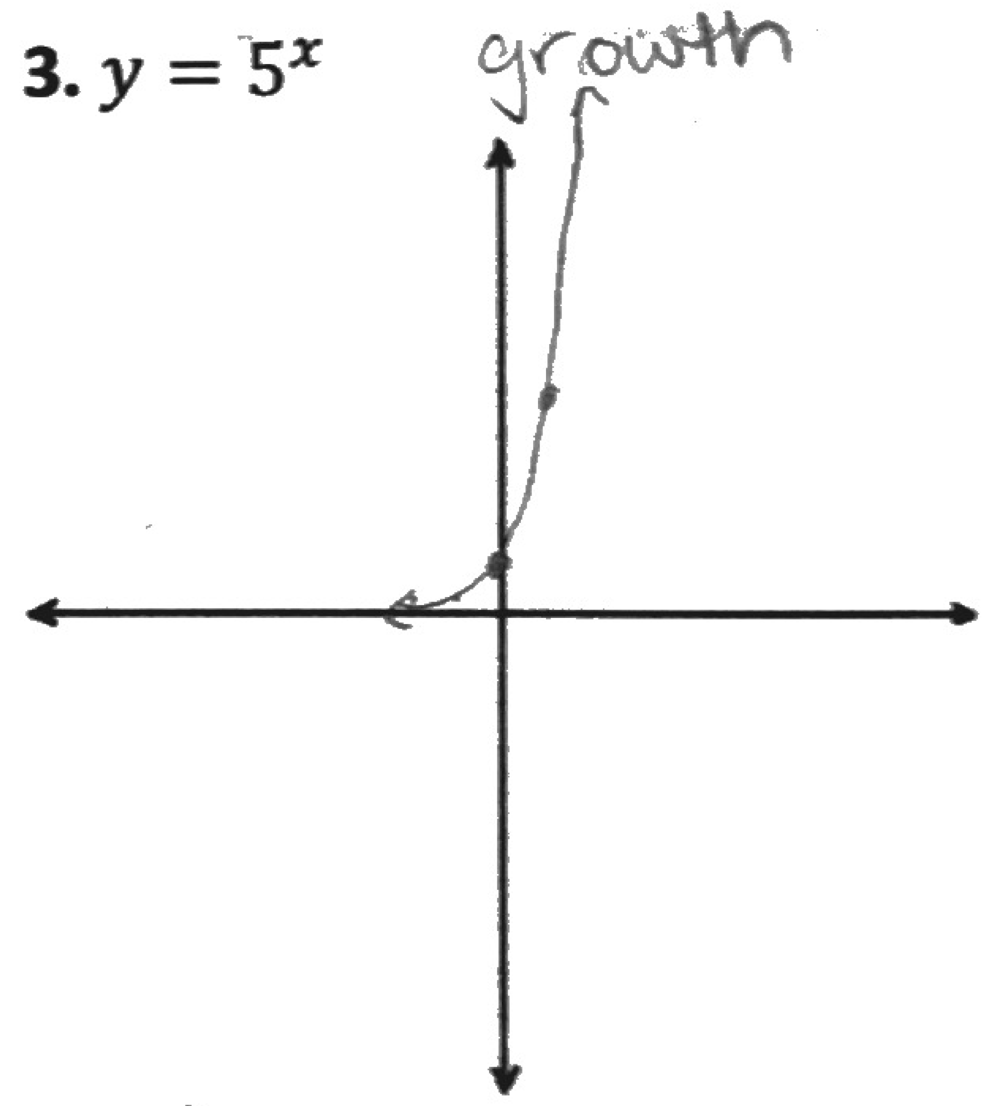
#2-4: Tell whether the function represents exponential growth or exponential decay. Identify the percent increase or decrease. Then graph the function.

2. $f(x) = \left(\frac{1}{3}\right)^x$ decay



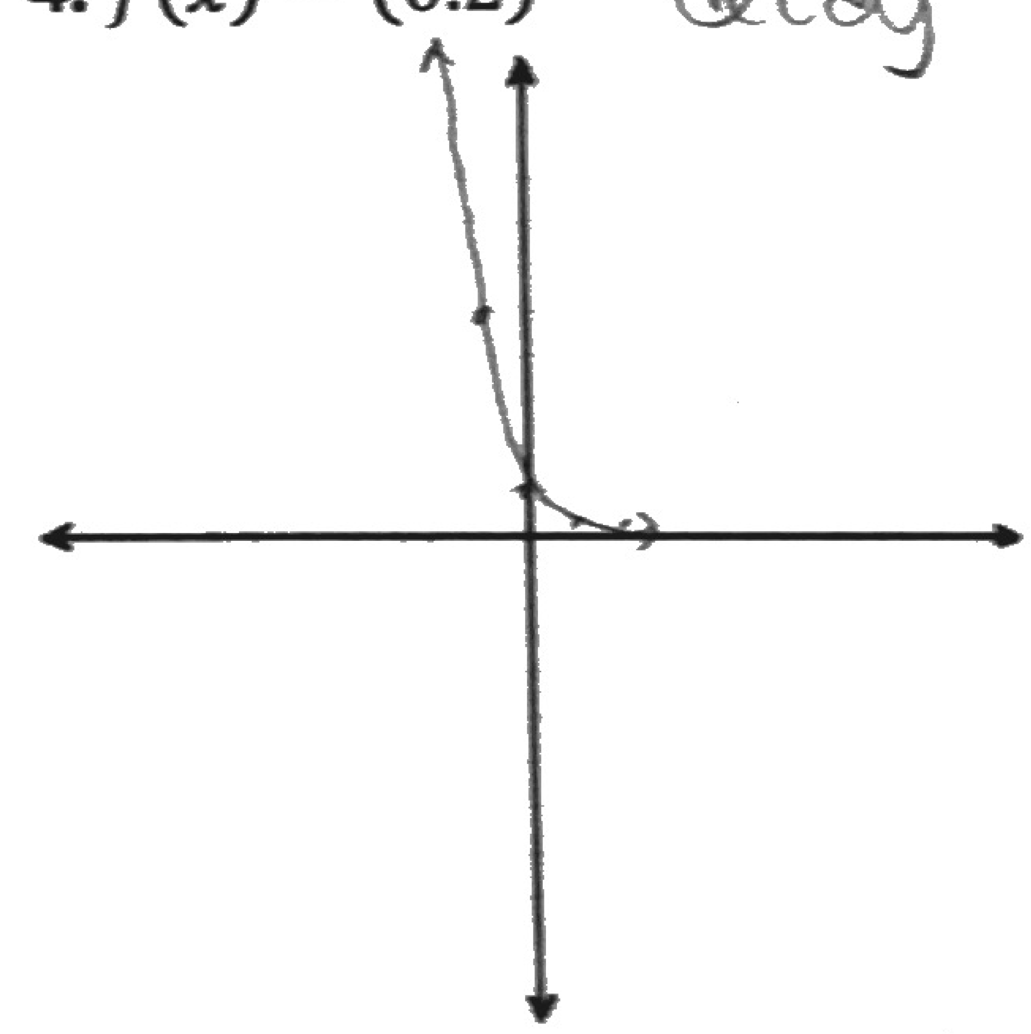
x	-2	-1	0	1	2
f(x)	9	3	1	1/3	1/9

3. $y = 5^x$ growth



x	-2	-1	0	1	2
y	1/25	1/5	1	5	25

4. $f(x) = (0.2)^x$ $= \left(\frac{1}{5}\right)^x$ decay



x	-2	-1	0	1	2
f(x)	25	5	1	1/5	1/25

5. You deposit $1500 in an account that pays 7% annual interest. Find the balance after 2 years when the interest is compounded daily.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ P = $1500 n = 365

r = 0.07 t = 2

$A = 1500\left(1 + \frac{0.07}{365}\right)^{365 \cdot 2}$

$\boxed{A = \$1725.39}$

7.2 The Natural Base *e*

#6-10: Simplify the expression.

6. $\frac{18e^{13}}{2e^7}$

$9e^{13-1}$

$\boxed{9e^6}$

7. $(2e^{3x})^3$

$2^3 e^{3x \cdot 3}$

$\boxed{8e^{9x}}$

8. $e^4 \cdot e^{11}$

e^{4+11}

$\boxed{e^{15}}$

9. $\frac{20e^3}{10e^6}$

$2e^{3-6}$

$2e^{-3}$

$\boxed{\frac{2}{e^3}}$

10. $(-3e^{-5x})^2$

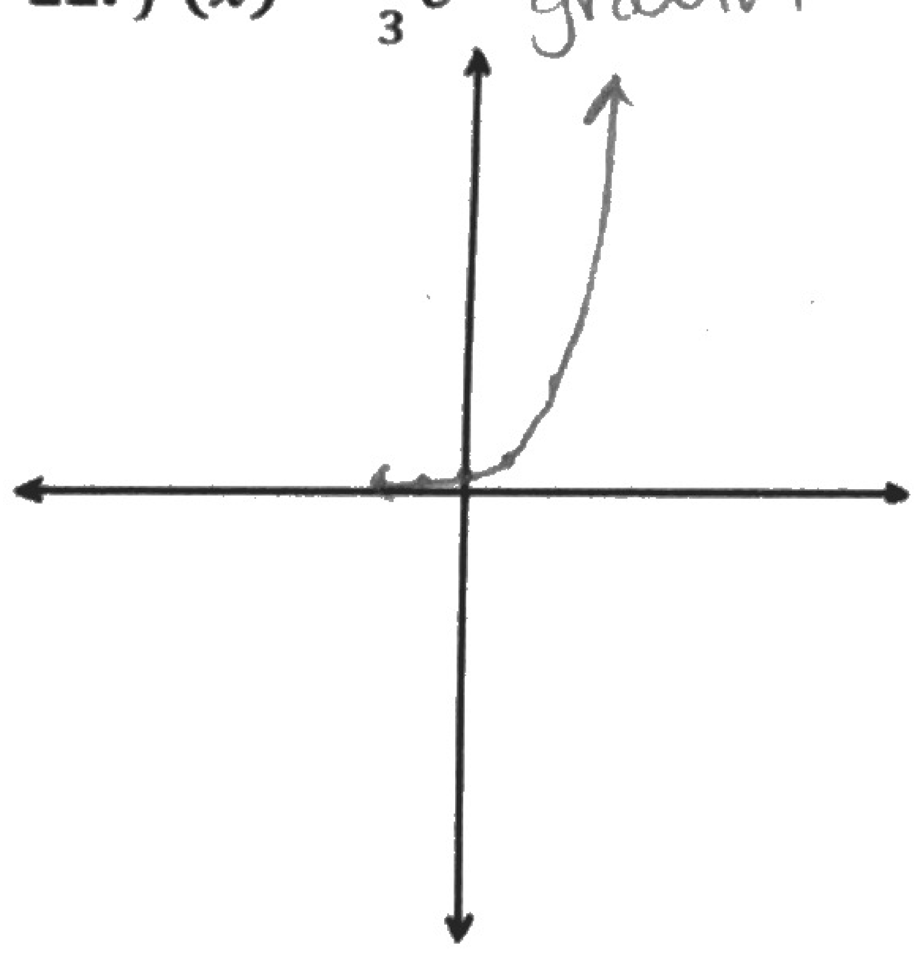
$(-3)^2 e^{-5x \cdot 2}$

$9e^{-10x}$

$\boxed{\frac{9}{e^{10x}}}$

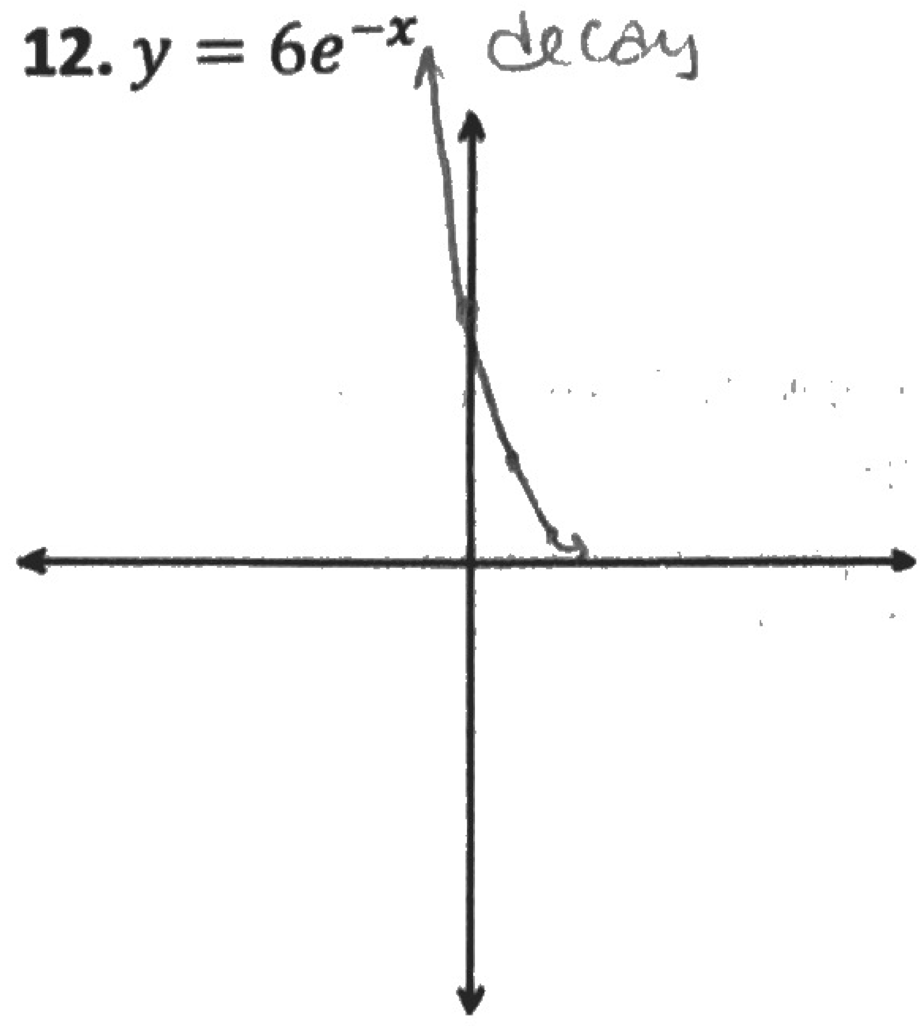
#11-13: Tell whether the function represents *exponential growth* or *exponential decay*. Then graph the function.

11. $f(x) = \frac{1}{3}e^x$ growth



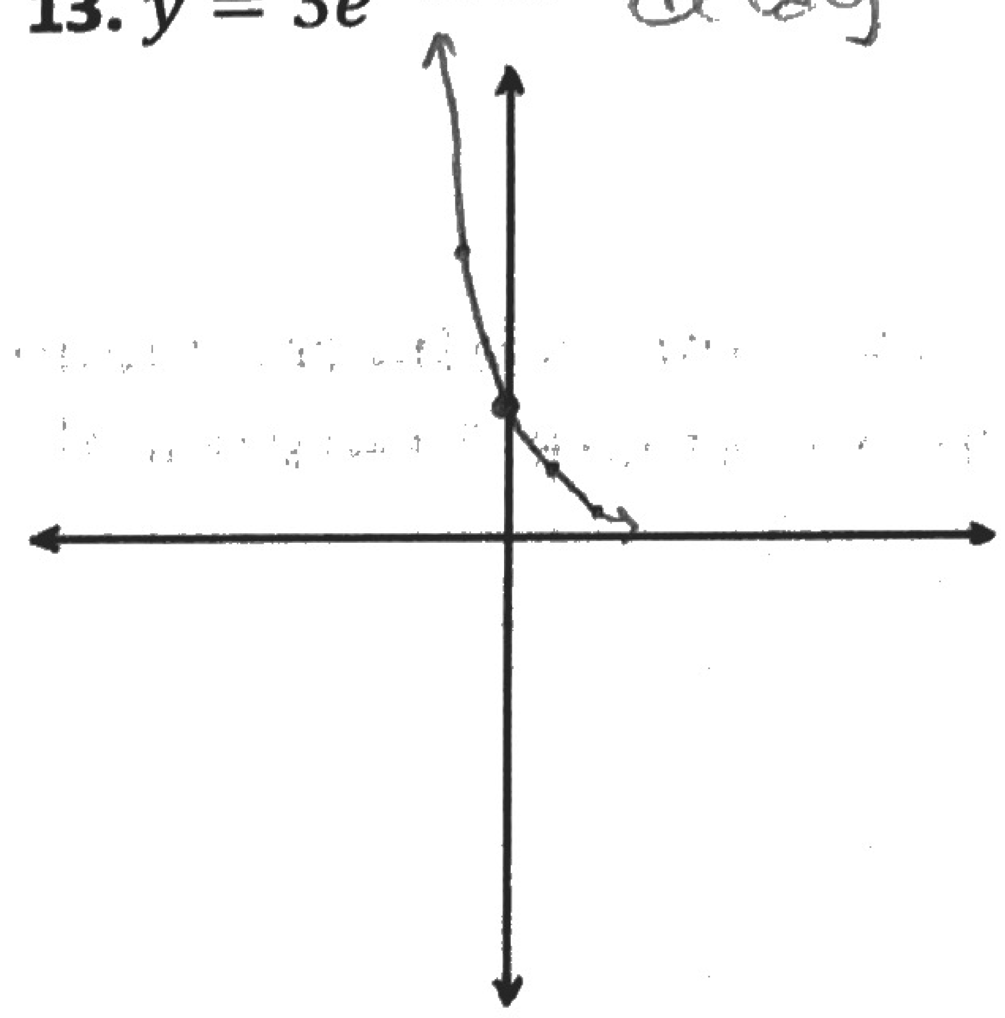
x	-2	-1	0	1	2
f(x)	$\frac{1}{3e^2}$	$\frac{1}{3e}$	1/3	$\frac{1}{3}e$	$\frac{1}{3}e^2$

12. $y = 6e^{-x}$ decay



x	-2	-1	0	1	2
y	$6e^2$	$6e$	6	$\frac{6}{e}$	$\frac{6}{e^2}$

13. $y = 3e^{-0.75x}$ decay



x	-2	-1	0	1	2
y	13.441	6.351	3	1.417	0.669

7.3 Logarithms and Logarithmic Functions

14. Find the inverse of function $y = \ln(x-2)$.

$x = \ln(y-2)$

$e^x = e^{\ln(y-2)}$

$e^x = y-2$

$\boxed{e^x + 2 = y}$

#15-17: Evaluate the logarithm.

15. $\log_2 8 = 3$

$2^? = 8$

16. $\log_6 \frac{1}{36} = -2$

$6^? = \frac{1}{36}$

17. $\log_5 1 = 0$

$5^? = 1$

#18-20: Find the inverse of the function.

18. $f(X) = 8^x$

$x = 8^y$

$\log_8 x = \log_8 8^y$

$\boxed{\log_8 x = y}$

19. $y = \ln(x-4)$

$x = \ln(y-4)$

$e^x = e^{\ln(y-4)}$

$e^x = y-4$

$\boxed{e^x + 4 = y}$

20. $y = \log(x+9)$

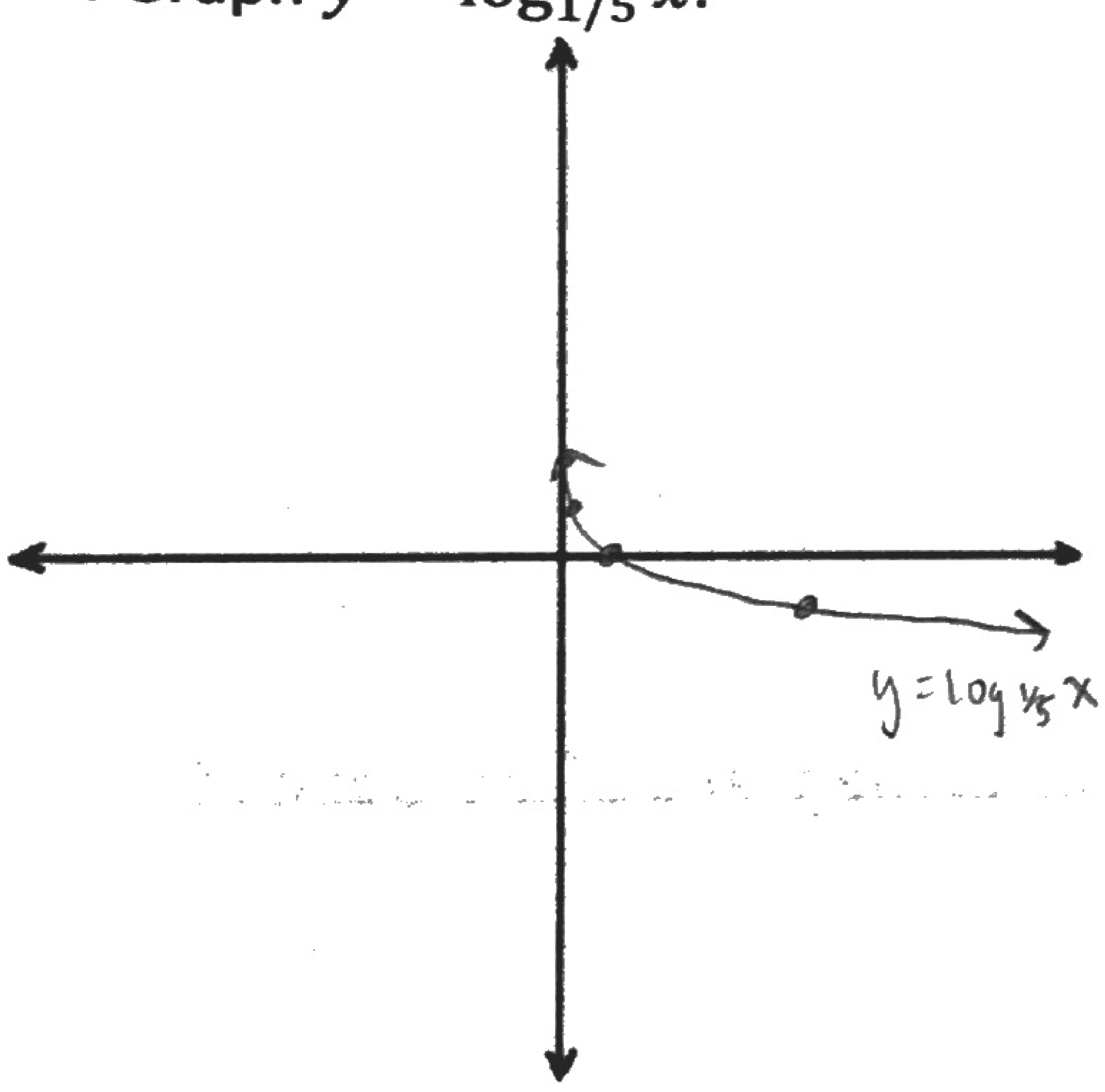
$x = \log(y+9)$

$10^x = 10^{\log(y+9)}$

$10^x = y+9$

$\boxed{10^x - 9 = y}$

21. Graph $y = \log_{1/5} x$.



inverse: $y = \left(\frac{1}{5}\right)^x$

x	y
-2	25
-1	5
0	1
1	1/5
2	1/25

$y = \log_{1/5} x$

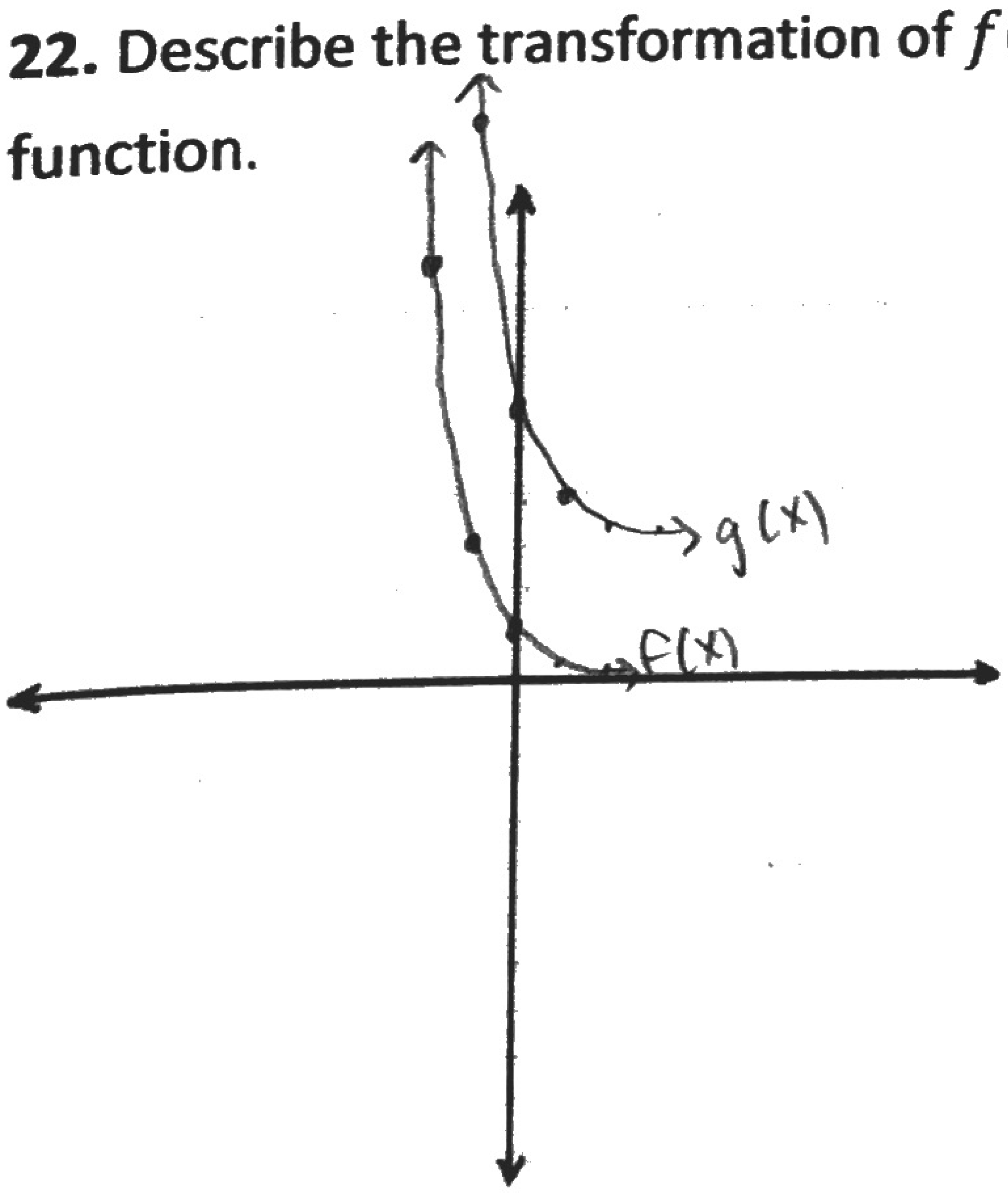
x	y
25	-2
5	-1
1	0
1/5	1
1/25	2

7.4 Transformations of Exponential and Logarithmic Functions

22. Describe the transformation of $f(x) = \left(\frac{1}{3}\right)^x$ represented by $g(x) = \left(\frac{1}{3}\right)^{x-1} + 3$. Then graph each function.

horizontal translation right 1 unit

vertical translation up 3 units



x	$f(x) = \left(\frac{1}{3}\right)^x$	$g(x) = \left(\frac{1}{3}\right)^{x-1}$
-2	9	30
-1	3	12
0	1	5
1	1/3	4
2	1/9	3 1/3

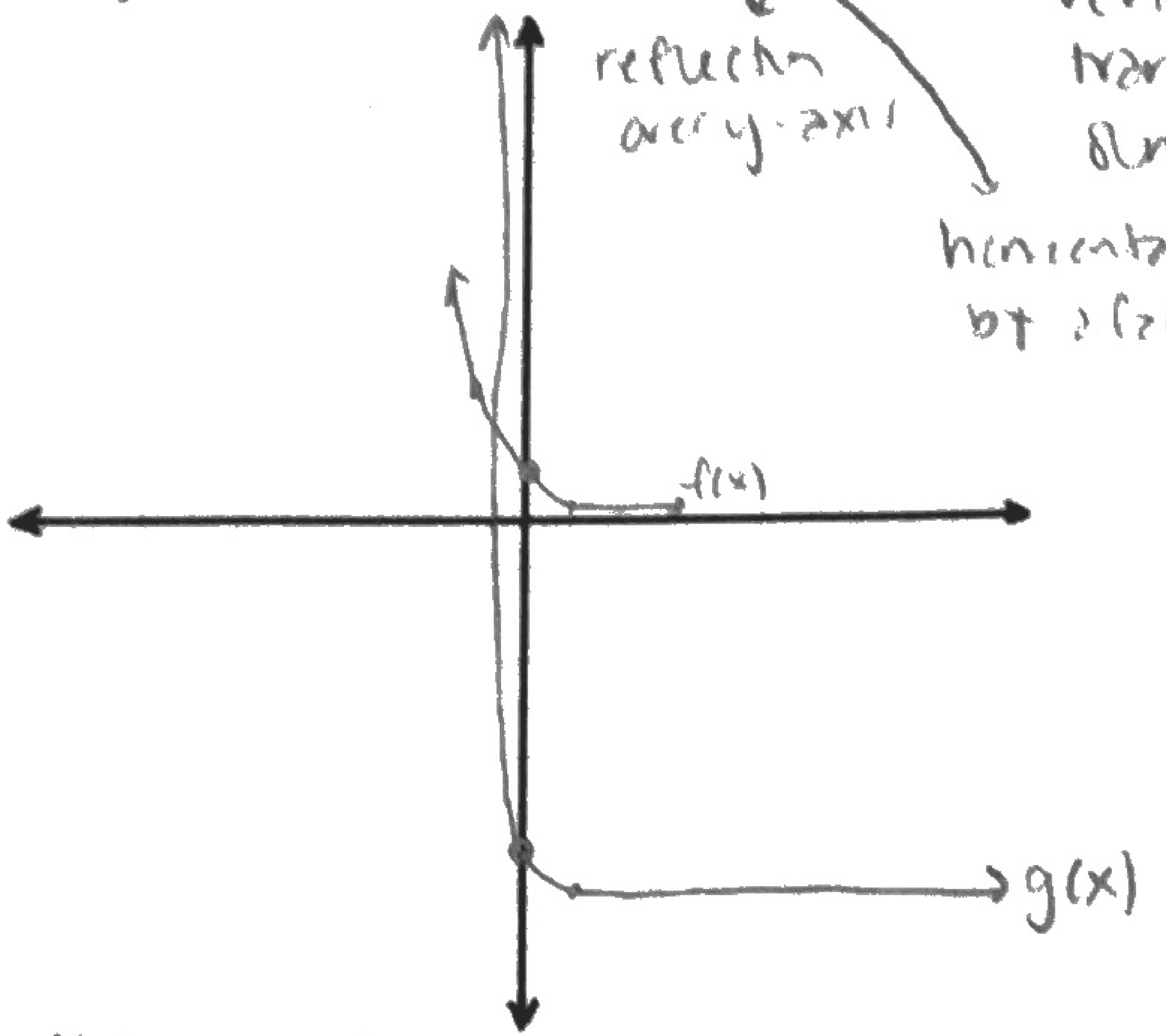
#23-24: Describe the transformation of f represented by g. Then graph each function.

23. $f(x) = e^{-x}, g(x) = e^{-5x} - 8$

reflection over y-axis

vertical translation 8 units down

horizontal shrink by a factor of 5

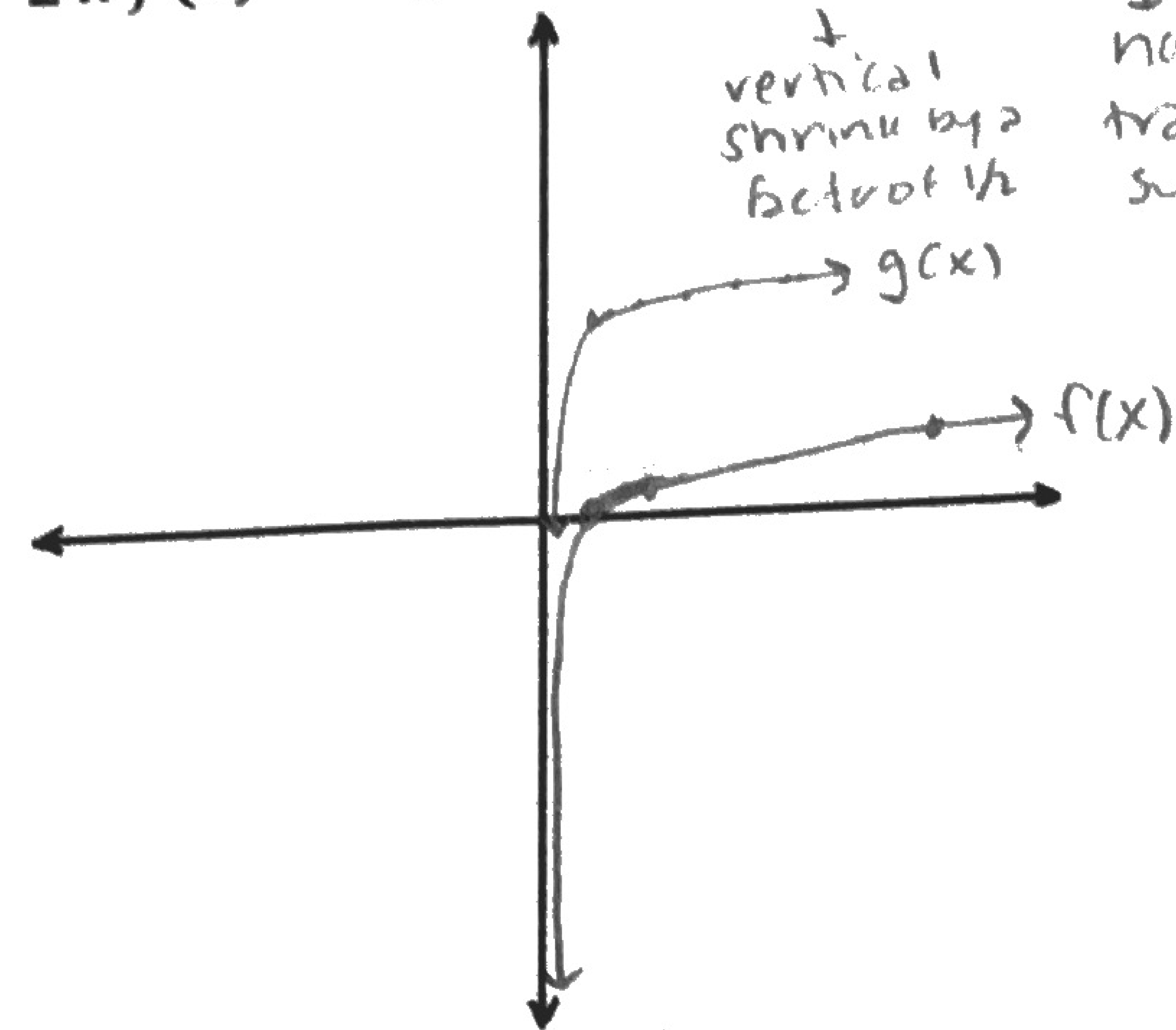


x	-2	-1	0	1	2
g(x)	22016	140	-7	-7.9	-7.9

24. $f(x) = \log_4 x, g(x) = \frac{1}{2}\log_4(x + 5)$

vertical shrink by a factor of 1/2

horizontal translation 5 units left



x	1	2	3	4	5
g(x)	4.1	4.4	4.5	4.6	4.7

#25-26: Write a rule for g.

25. Let the graph of g be a vertical stretch by a factor of 3, followed by a translation 6 units left and 3 units up of the graph of $f(x) = e^x$

1 vertical stretch by a factor of 3

$h(x) = 3f(x)$

$h(x) = 3e^x$

2 translation 6 units left and 3 units up

$g(x) = h(x+6) + 3$

$\boxed{g(x) = 3e^{x+6} + 3}$

26. Let the graph of g be a translation 2 units down, followed by a reflection in the y-axis of the graph of $f(x) = \log x$.

1 translation 2 units down

$h(x) = f(x) - 2$

$h(x) = \log x - 2$

2 reflection in y-axis

$g(x) = h(-x)$

$\boxed{g(x) = \log(-x) - 2}$

7.5 Properties of Logarithms

27. Expand $\ln \frac{12x^5}{y}$.

$(\ln 12 + \ln x^5) - \ln y$

$\boxed{(\ln 12 + 5\ln x) - \ln y}$

#28-33: Expand or condense the logarithmic expression.

28. $\log_8 3xy$

$\boxed{\log_8 3+\log_8 x+\log_8 y}$

29. $\log 10x^3y$

$\log 10+\log x^3+\log y$

$\boxed{\log 10+3\log x+\log y}$

30. $\ln\frac{3y}{x^5}$

$(\ln 3+\ln y)-\ln x^5$

$\boxed{(\ln 3+\ln y)-5\ln x}$

31. $3\log_7 4+\log_2 12$

$\log_7 4^3+\log_2 12$

$\log_7(4^3\cdot 12)$

$\boxed{\log_7 768}$

32. $\log_2 12-2\log_2 x$

$\log_2 12-\log_2 x^2$

$\boxed{\log_2\frac{12}{x^2}}$

33. $2\ln x+5\ln 2-\ln 8$

$\ln x^2+\ln 2^5-\ln 8$

$\ln(x^2\cdot 2^5)-\ln 8$

$\ln\frac{(x^2\cdot 2^5)}{8}$

$\ln\frac{32x^2}{8}$

$\boxed{\ln 4x^2}$

#34-36: Use the change-of-base formula to evaluate the logarithm.

34. $\log_2 10$

$\frac{\log 10}{\log 2}\approx\boxed{3.322}$

35. $\log_7 9$

$\frac{\log 9}{\log 7}\approx\boxed{1.129}$

36. $\log_{23} 42$

$\frac{\log 42}{\log 23}\approx\boxed{1.192}$

7.6 Solving Exponential and Logarithmic Equations

37. Solve $\ln(3x-9)=\ln(2x+6)$.

$e^{\ln(3x-9)}=e^{\ln(2x+6)}$

$3x-9=2x+6$

$\boxed{x=15}$

#38-40: Solve the equation. Check for extraneous solutions.

38. $5^x = 8$

$\log_5 5^x = \log_5 8$

$\boxed{x = \log_5 8}$

39. $\log_3(2x - 5) = 2$

$3^{\log_3(2x-5)} = 3^2$

$2x - 5 = 9$

$2x = 14$

$\boxed{x = 7}$

40. $\ln x + \ln(x + 2) = 3$

$\ln(x(x+2)) = 3$

$\ln(x^2+2x) = 3$

$e^{\ln(x^2+2x)} = e^3$

$x^2 + 2x = e^3$

$x^2 + 2x - e^3 = 0 \rightarrow$ solve using quad formula or graphing

$\boxed{x = 3.592}$

#41-43: Solve the inequality. (Not on assessment)

~~41.~~ $6^x > 12$

$\log_6 6^x > \log_6 12$

$x > \log_6 12$

$\boxed{x > 1.387}$

~~42.~~ $\ln x \leq 9$

$e^{\ln x} \leq e^9$

$x \leq e^9$

$\boxed{0 < x \leq 8103.084}$

~~43.~~ $e^{4x-2} \geq 16$

$\ln e^{4x-2} \geq \ln 16$

$4x - 2 \geq \ln 16$

$4x \geq \ln 16 + 2$

$x \geq \frac{\ln 16 + 2}{4}$

$\boxed{x \geq 1.193...}$

7.7 Modeling with Exponential and Logarithmic Functions

44. Write an exponential function whose graph passes through (1,3) and (4,24).

$y = ab^x$

$3 = ab^1$

$\frac{3}{b} = a$

$a = \frac{3}{2}$

$24 = ab^4$

$24 = \left(\frac{3}{b}\right) b^4$

$24 = 3b^3$

$8 = b^3$

$2 = b$

$\boxed{y = \frac{3}{2}(2^x)}$

#45-46: Write an exponential model for the data pairs (x, y).

45. $(3,8), (5,2)$

$y = ab^x$

$8 = ab^3$ $\quad$ $2 = ab^5$

$\frac{8}{b^3} = a$ $\quad$ $2 = \left(\frac{8}{b^3}\right)b^5$

$a = \frac{8}{\left(\frac{1}{2}\right)^3}$ $\quad$ $2 = 8b^2$

$a = 64$ $\quad$ $\frac{1}{4} = b^2$

$\frac{1}{2} = b$

$\boxed{y = 64\left(\frac{1}{2}\right)^x}$

46.

x	1	2	3	4
$\ln y$	1.64	2.00	2.36	2.72

+0.36 $\quad$ +0.36 $\quad$ +0.36

$\ln y - \ln y_1 = m(x - x_1)$

$\ln y - 1.64 = 0.36(x - 1)$

$\ln y = 0.36x + 1.28$

$y = e^{0.36x + 1.28}$

$y = e^{0.36x} \cdot e^{1.28}$

$\boxed{y = 3.60(1.43)^x}$

A shoe store sells a new type of basketball shoe. The table shows the pairs sold s over time t (in weeks). Use a graphing calculator to find a logarithmic model of the form $s = a + b\ln t$ that represents the data. Estimate how many pairs of shoes are sold after 6 weeks. Not on assessment - need graphing calc to do logarithmic regression curves.

Week, t	1	3	5	7	9
Pairs sold, s	5	32	48	58	65

Enter data into graphing calculator and perform logarithmic regression.

The model is $s = 3.9521 + 27.475\ln t$

Substitute $t = 6$ into the model to obtain $s \approx 53$.

So 53 pairs of shoes are sold after 6 weeks.