7.4 Transformations of Exponential and Logarithmic Functions Homework

#3-6, 17-24 odd (do not graph), 25, 26, 27-30 odd (do not graph), 31-34, 35-42 odd, 45-48, 50-55

Vocabulary and Core Concept Check

1. **WRITING** Given the function $f(x) = ab^{x-h} + k$, describe the effects of a, h, and k on the graph of the function.

2. **COMPLETE THE SENTENCE** The graph of $g(x) = \log_4(-x)$ is a reflection in the _____ of the graph of $f(x) = \log_4 x$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, match the function with its graph. Explain your reasoning.

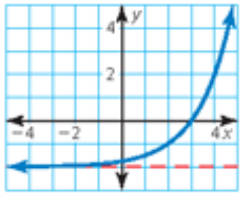
3. $f(x) = 2^{x+2} - 2$

4. $g(x) = 2^{x+2} + 2$

5. $h(x) = 2^{x-2} - 2$

6. $k(x) = 2^{x-2} + 2$

A.



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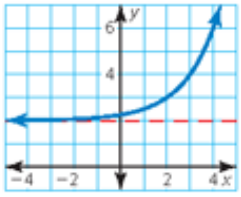

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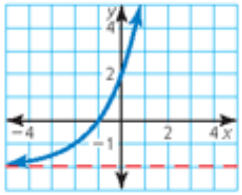

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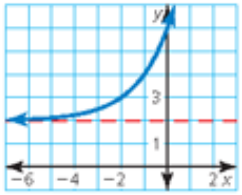


In Exercises 7–16, describe the transformation of f represented by g. Then graph each function. *(See Examples 1 and 2.)*

7. $f(x) = 3^x$, $g(x) = 3^x + 5$

8. $f(x) = 4^x$, $g(x) = 4^x - 8$

9. $f(x) = e^x$, $g(x) = e^x - 1$

10. $f(x) = e^x$, $g(x) = e^x + 4$

11. $f(x) = 2^x$, $g(x) = 2^{x-7}$

12. $f(x) = 5^x$, $g(x) = 5^{x+1}$

13. $f(x) = e^{-x}$, $g(x) = e^{-x} + 6$

14. $f(x) = e^{-x}$, $g(x) = e^{-x} - 9$

15. $f(x) = \left(\frac{1}{4}\right)^x$, $g(x) = \left(\frac{1}{4}\right)^{x-3} + 12$

16. $f(x) = \left(\frac{1}{3}\right)^x$, $g(x) = \left(\frac{1}{3}\right)^{x+2} - \frac{2}{3}$

In Exercises 17–24, describe the transformation of f represented by g. Then graph each function. *(See Example 3.)*

17. $f(x) = e^x$, $g(x) = e^{2x}$

18. $f(x) = e^x$, $g(x) = \frac{4}{3}e^x$

19. $f(x) = 2^x$, $g(x) = -2^{x-3}$

20. $f(x) = 4^x$, $g(x) = 4^{0.5x-5}$

21. $f(x) = e^{-x}$, $g(x) = 3e^{-6x}$

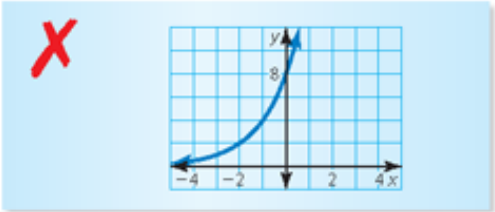
22. $f(x) = e^{-x}$, $g(x) = e^{-5x} + 2$

23. $f(x) = \left(\frac{1}{2}\right)^x$, $g(x) = 6\left(\frac{1}{2}\right)^{x+5} - 2$

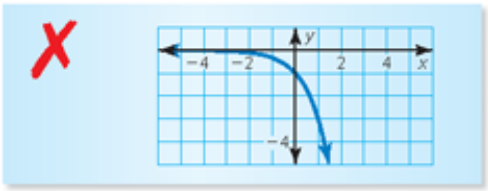
24. $f(x) = \left(\frac{3}{4}\right)^x$, $g(x) = -\left(\frac{3}{4}\right)^{x-7} + 1$

ERROR ANALYSIS **In Exercises 25 and 26, describe and correct the error in graphing the function.**

25. $f(x) = 2^x + 3$



26. $f(x) = 3^{-x}$



In Exercises 27–30, describe the transformation of f represented by g. Then graph each function. *(See Example 4.)*

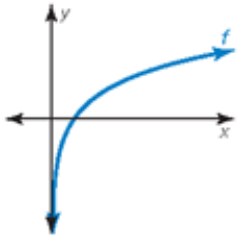
27. $f(x) = \log_4 x$, $g(x) = 3 \log_4 x - 5$

28. $f(x) = \log_{1/3} x$, $g(x) = \log_{1/3}(-x) + 6$

29. $f(x) = \log_{1/5} x$, $g(x) = -\log_{1/5}(x - 7)$

30. $f(x) = \log_2 x$, $g(x) = \log_2(x + 2) - 3$

ANALYZING RELATIONSHIPS In Exercises 31–34, match the function with the correct transformation of the graph of f. Explain your reasoning.



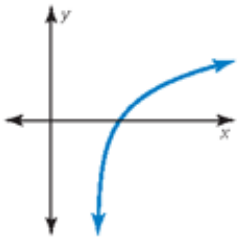
31. $y = f(x - 2)$

32. $y = f(x + 2)$

33. $y = 2f(x)$

34. $y = f(2x)$

A.



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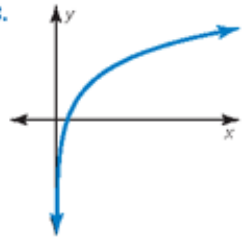

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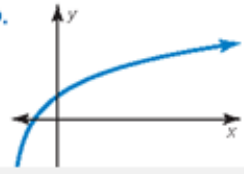


In Exercises 35–38, write a rule for g that represents the indicated transformations of the graph of f. *(See Example 5.)*

35. $f(x) = 5^x$; translation 2 units down, followed by a reflection in the y-axis

36. $f(x) = \left(\frac{2}{3}\right)^x$; reflection in the x-axis, followed by a vertical stretch by a factor of 6 and a translation 4 units left

37. $f(x) = e^x$; horizontal shrink by a factor of $\frac{1}{2}$, followed by a translation 5 units up

38. $f(x) = e^{-x}$; translation 4 units right and 1 unit down, followed by a vertical shrink by a factor of $\frac{1}{3}$

In Exercises 39–42, write a rule for g that represents the indicated transformation of the graph of f. *(See Example 6.)*

39. $f(x) = \log_6 x$; vertical stretch by a factor of 6, followed by a translation 5 units down

40. $f(x) = \log_5 x$; reflection in the x-axis, followed by a translation 9 units left

41. $f(x) = \log_{1/2} x$; translation 3 units left and 2 units up, followed by a reflection in the y-axis

42. $f(x) = \ln x$; translation 3 units right and 1 unit up, followed by a horizontal stretch by a factor of 8

JUSTIFYING STEPS In Exercises 43 and 44, justify each step in writing a rule for g that represents the indicated transformations of the graph of f.

43. $f(x) = \log_7 x$; reflection in the x-axis, followed by a translation 6 units down

$h(x) = -f(x)$ _____

$= -\log_7 x$ _____

$g(x) = h(x) - 6$ _____

$= -\log_7 x - 6$ _____

44. $f(x) = 8^x$; vertical stretch by a factor of 4, followed by a translation 1 unit up and 3 units left

$h(x) = 4 \cdot f(x)$ _____

$= 4 \cdot 8^x$ _____

$g(x) = h(x + 3) + 1$ _____

$= 4 \cdot 8^{x+3} + 1$ _____

USING STRUCTURE **In Exercises 45–48, describe the transformation of the graph of f represented by the graph of g. Then give an equation of the asymptote.**

45. $f(x) = e^x$, $g(x) = e^x + 4$

46. $f(x) = 3^x$, $g(x) = 3^{x-9}$

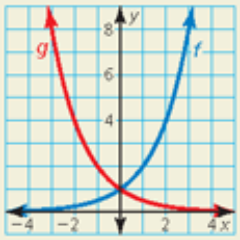
47. $f(x) = \ln x$, $g(x) = \ln(x + 6)$

48. $f(x) = \log_{1/5} x$, $g(x) = \log_{1/5} x + 13$

49. **MODELING WITH MATHEMATICS** The slope S of a beach is related to the average diameter d (in millimeters) of the sand particles on the beach by the equation $S = 0.159 + 0.118 \log d$. Describe the transformation of $f(d) = \log d$ represented by S. Then use the function to determine the slope of a beach for each sand type below.

Sand particle	Diameter (mm), d
fine sand	0.125
medium sand	0.25
coarse sand	0.5
very coarse sand	1

50. **HOW DO YOU SEE IT?** The graphs of $f(x) = b^x$ and $g(x) = \left(\frac{1}{b}\right)^x$ are shown for $b = 2$.



a. Use the graph to describe a transformation of the graph of f that results in the graph of g.

b. Does your answer in part (a) change when $0 < b < 1$? Explain.

51. **MAKING AN ARGUMENT** Your friend claims a single transformation of $f(x) = \log x$ can result in a function g whose graph never intersects the graph of f. Is your friend correct? Explain your reasoning.

52. **THOUGHT PROVOKING** Is it possible to transform the graph of $f(x) = e^x$ to obtain the graph of $g(x) = \ln x$? Explain your reasoning.

53. **ABSTRACT REASONING** Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

a. A vertical translation of the graph of $f(x) = \log x$ changes the equation of the asymptote.

b. A vertical translation of the graph of $f(x) = e^x$ changes the equation of the asymptote.

c. A horizontal shrink of the graph of $f(x) = \log x$ does not change the domain.

d. The graph of $g(x) = ab^{x-h} + k$ does not intersect the x-axis.

54. **PROBLEM SOLVING** The amount P (in grams) of 100 grams of plutonium-239 that remains after t years can be modeled by $P = 100(0.99997)^t$.

a. Describe the domain and range of the function.

b. How much plutonium-239 is present after 12,000 years?

c. Describe the transformation of the function if the initial amount of plutonium were 550 grams.

d. Does the transformation in part (c) affect the domain and range of the function? Explain your reasoning.

55. **CRITICAL THINKING** Consider the graph of the function $h(x) = e^{-x-2}$. Describe the transformation of the graph of $f(x) = e^{-x}$ represented by the graph of h. Then describe the transformation of the graph of $g(x) = e^x$ represented by the graph of h. Justify your answers.

56. **OPEN-ENDED** Write a function of the form $y = ab^{x-h} + k$ whose graph has a y-intercept of 5 and an asymptote of $y = 2$.