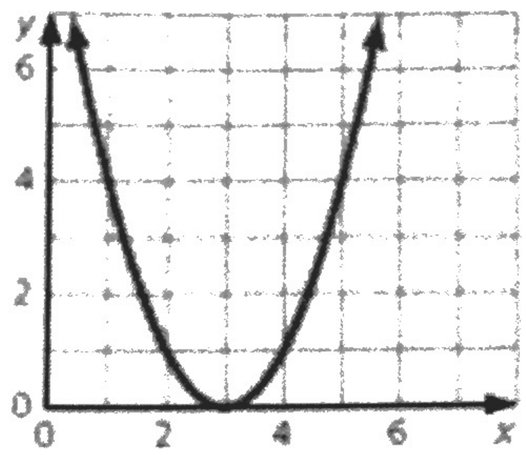
Name: Key Date: Band:
Algebra 2

Algebra 2 Practice Midterm

Functions

Find the domain and range of the function represented by the graph.

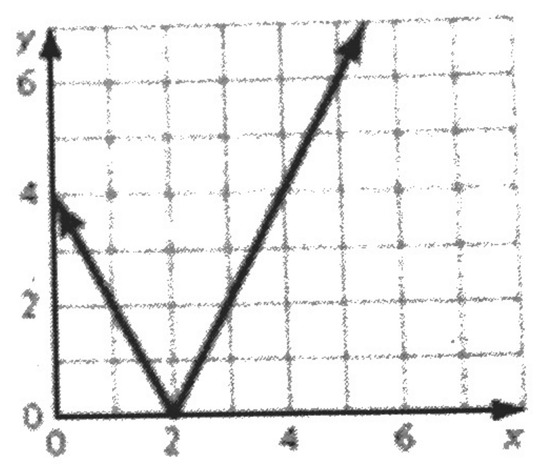
1.



domain: all real #s

range: $y \geq 0$ or $[0, \infty)$

2.



domain: all real #s

range: $y \geq 0$ or $[0, \infty)$

Evaluate the function when $x = -2, 0$, and 5.

3. $f(x) = x^2 + 6$

x	$f(x) = x^2 + 6$
-2	$f(-2) = (-2)^2 + 6 = 10$
0	$f(0) = (0)^2 + 6 = 6$
5	$f(5) = (5)^2 + 6 = 31$

4. $f(x) = -2x + 9$

x	$f(x) = -2x + 9$
-2	$f(-2) = -2(-2) + 9 = 13$
0	$f(0) = -2(0) + 9 = 9$
5	$f(5) = -2(5) + 9 = -1$

5. $f(x) = -3 + 4x^3$

x	$f(x) = -3 + 4x^3$
-2	$f(-2) = -3 + 4(-2)^3 = -35$
0	$f(0) = -3 + 4(0)^3 = -3$
5	$f(5) = -3 + 4(5)^3 = 497$

Find the value of x so that the function has the given value.

6. $m(x) = 4x^2 + 15; m(x) = 31$

$31 = 4x^2 + 15$

$16 = 4x^2$

$4 = x^2$

$\boxed{\pm 2 = x}$

7. $k(x) = 6x - 12; k(x) = 18$

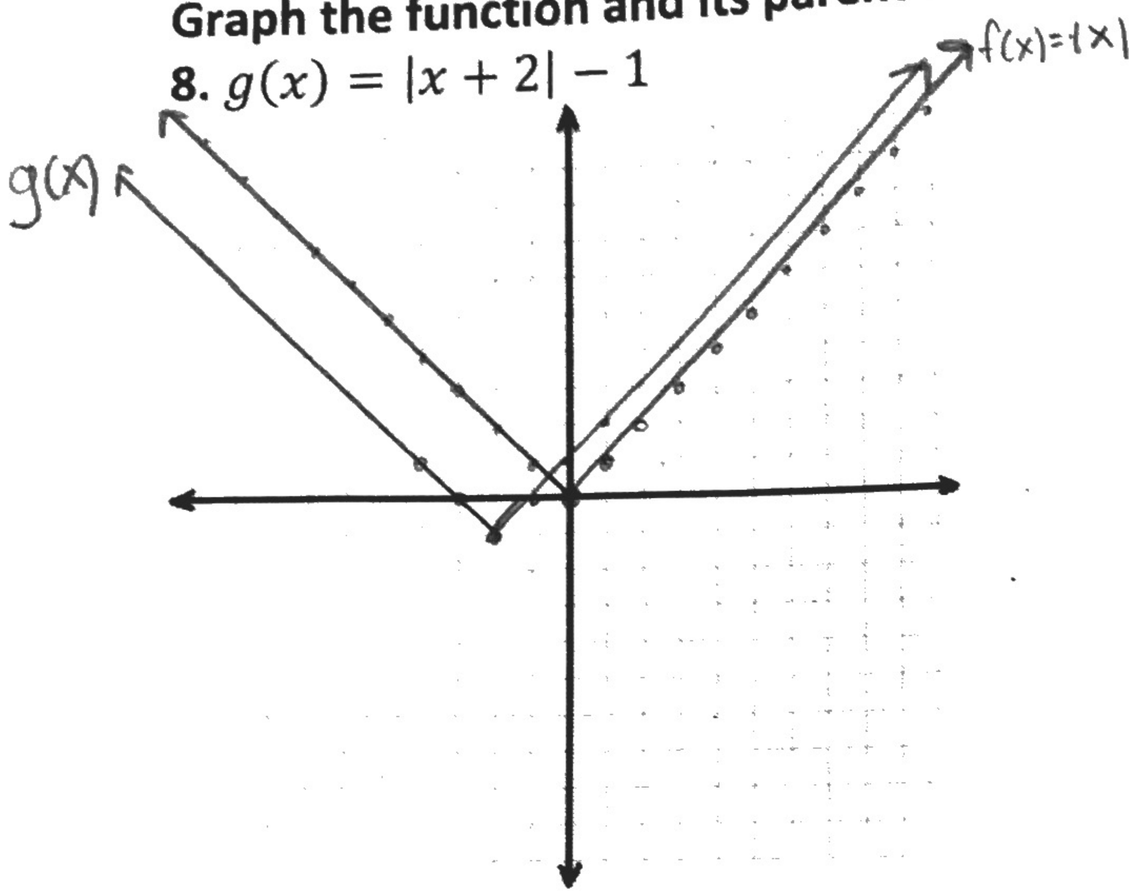
$18 = 6x - 12$

$30 = 6x$

$\boxed{5 = x}$

Graph the function and its parent function. Then describe the transformation.

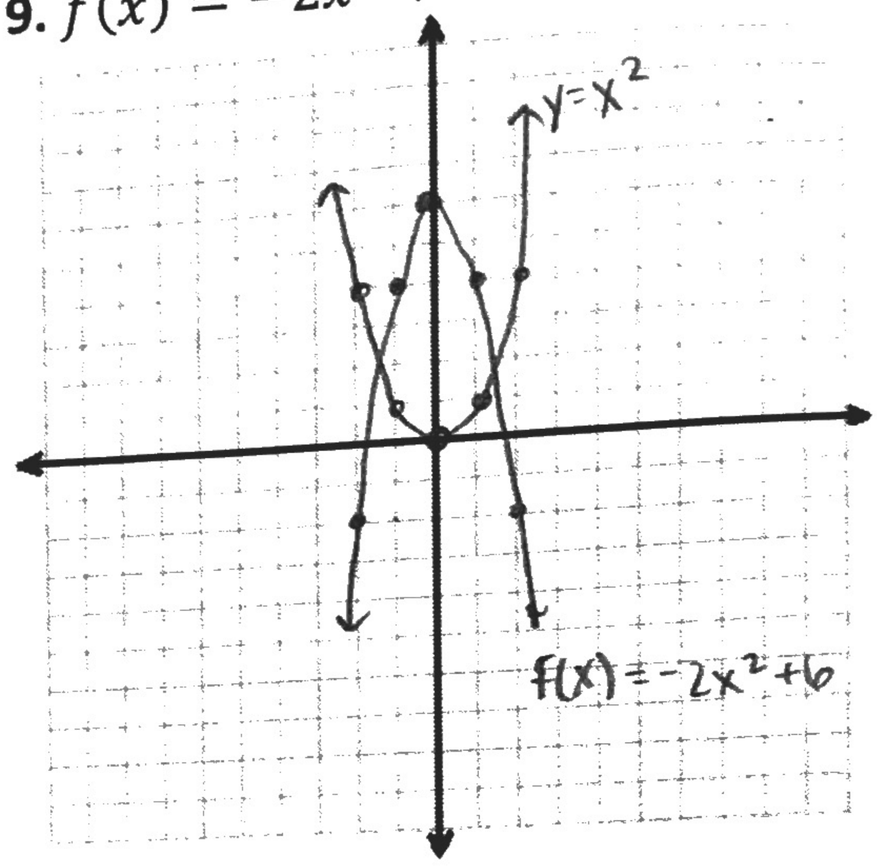
8. $g(x) = |x + 2| - 1$



horizontal translation left 2 units

vertical translation down 1 unit

9. $f(x) = -2x^2 + 6$



x	f(x)
-2	-2
-1	4
0	6
1	4
2	-2

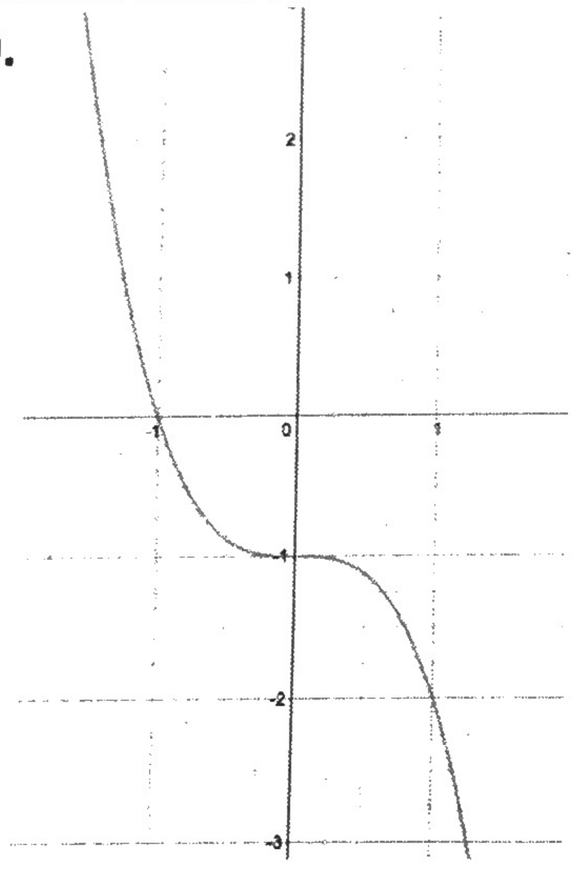
reflection over x-axis

vertical stretch by a factor of 2.

vertical translation up 6 units

Identify the parent function of the graph. Then write a function that represents its transformation.

10.



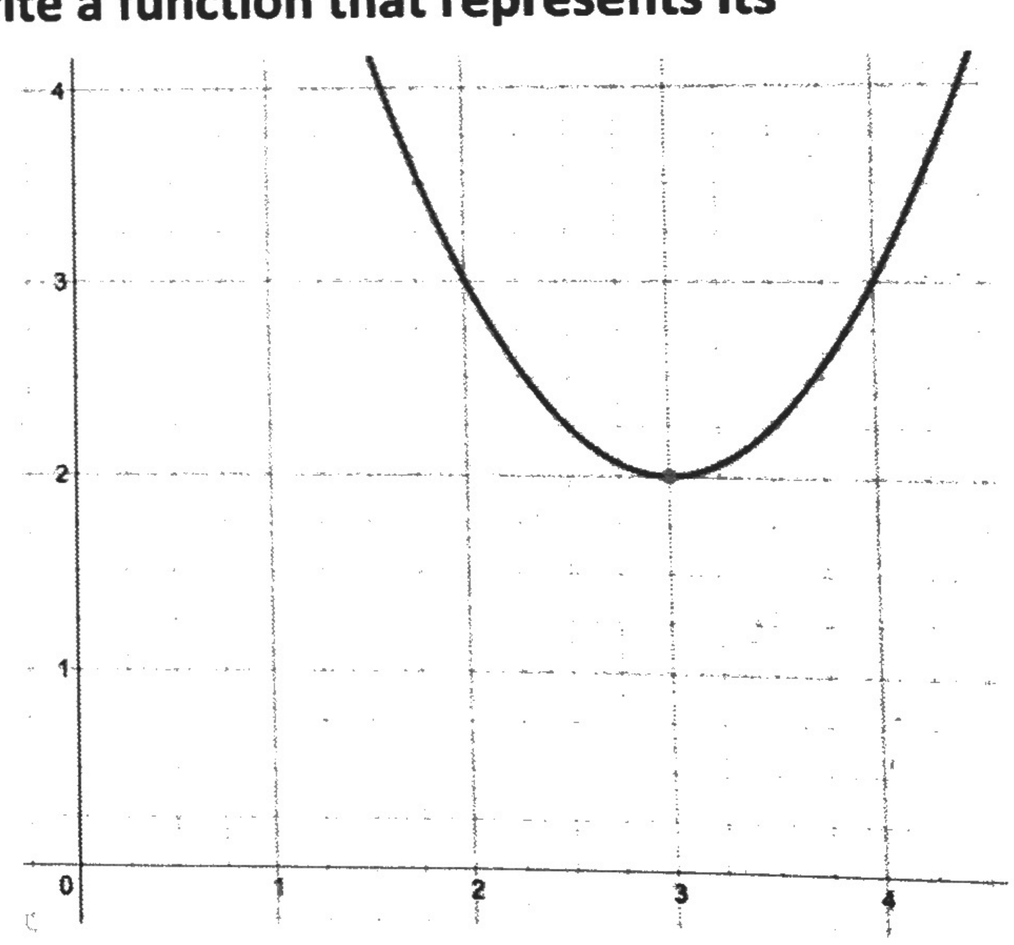
cubic: $y = x^3$

$f(x) = -x^3 - 1$

OR

$f(x) = (-x)^3 - 1$

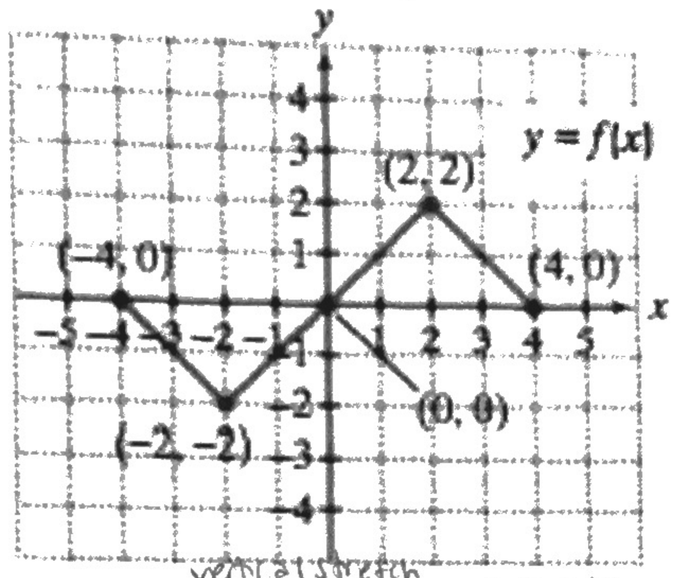
11.



quadratic: $y = x^2$

$f(x) = (x-3)^2 + 2$

Use the graph of $y = f(x)$ to graph each function g.

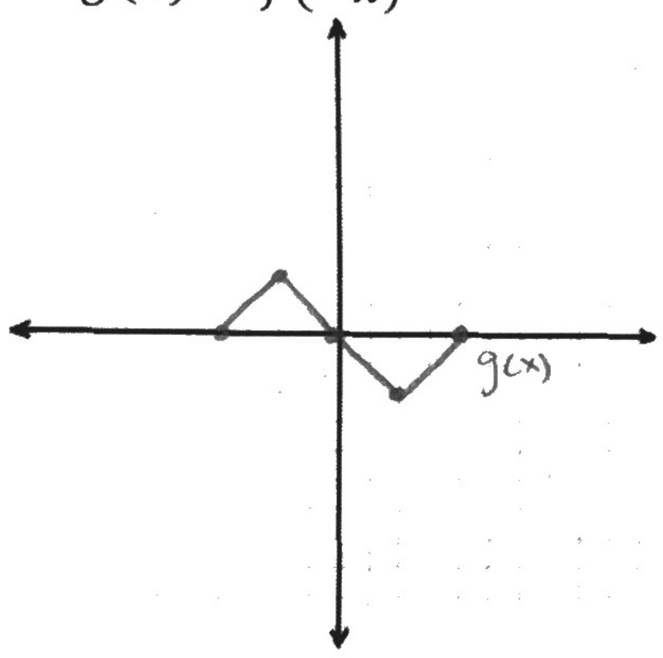


(13) $(-4,0) \rightarrow (-4,0)$
$(-2,-2) \rightarrow (-2,-4)$
$(0,0) \rightarrow (0,0)$
$(2,2) \rightarrow (2,4)$
$(4,0) \rightarrow (4,0)$

2f(x)

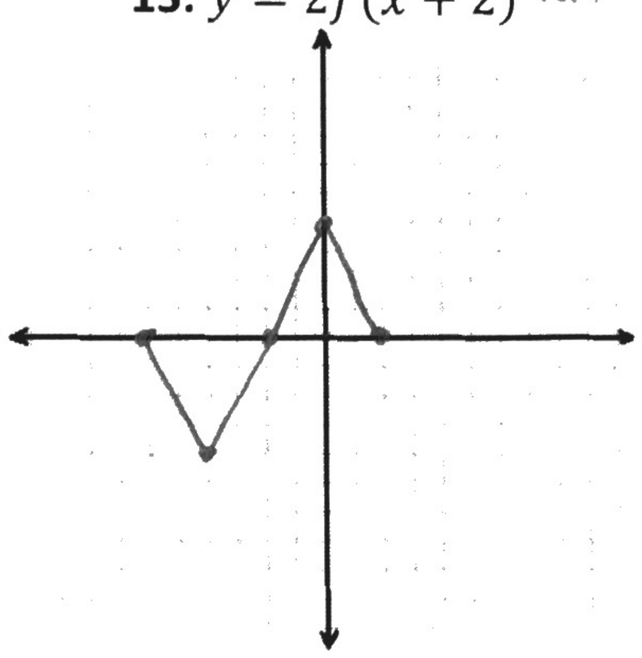
reflection over y-axis

12. $g(x) = f(-x)$



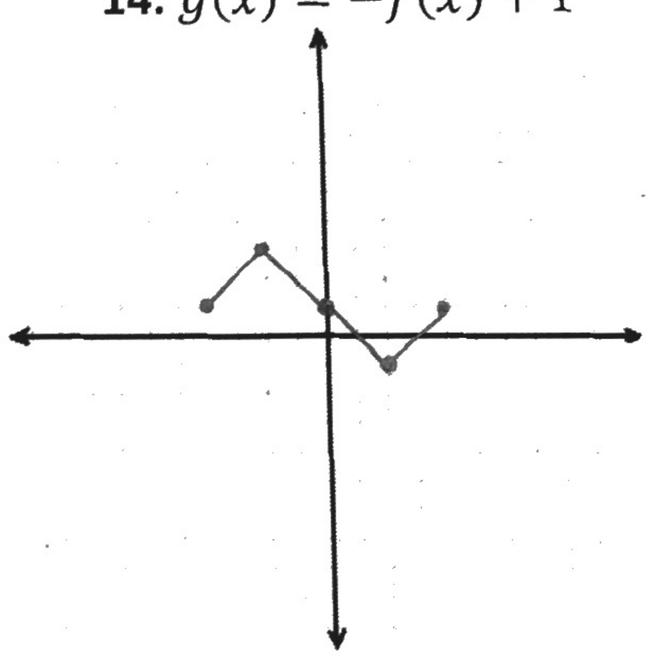
vertical stretch, 2 units left

13. $y = 2f(x+2)$



reflection over x-axis, 1 unit up

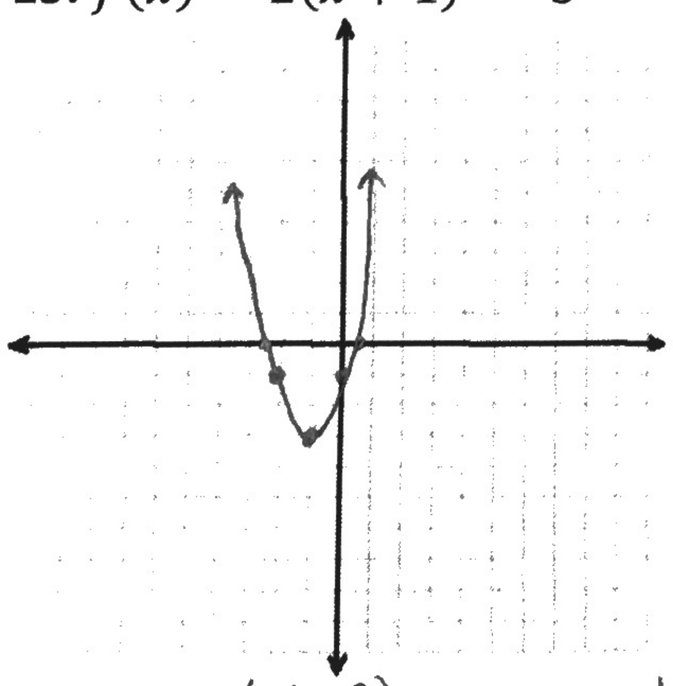
14. $g(x) = -f(x) + 1$



Graphing Quadratic Functions

Find the vertex, axis of symmetry, y-intercept, and x-intercepts of each function. Then graph the function.

15. $f(x) = 2(x+1)^2 - 3$



vertex: $(-1,-3)$ aos: $x = -1$

y-int: $f(0) = 2(0+1)^2 - 3 = -1$

x-int: $0 = 2(x+1)^2 - 3$

$3 = 2(x+1)^2$

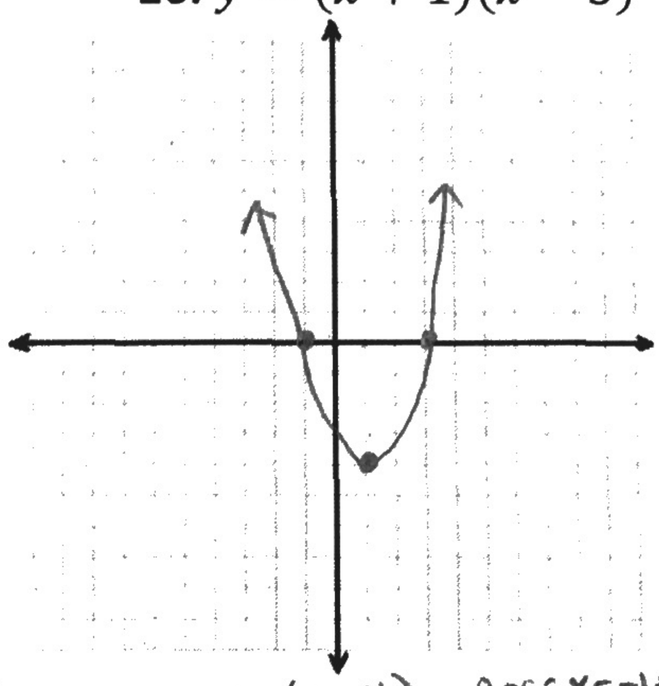
$\frac{3}{2} = (x+1)^2$

$\pm\sqrt{\frac{3}{2}} = x+1$

$x = -1 \pm \sqrt{\frac{3}{2}}$

$x = -1 \pm \frac{\sqrt{6}}{2}$

16. $y = (x+1)(x-3)$



vertex: $(1,-4)$ aos: $x = \frac{-1+3}{2} = 1$

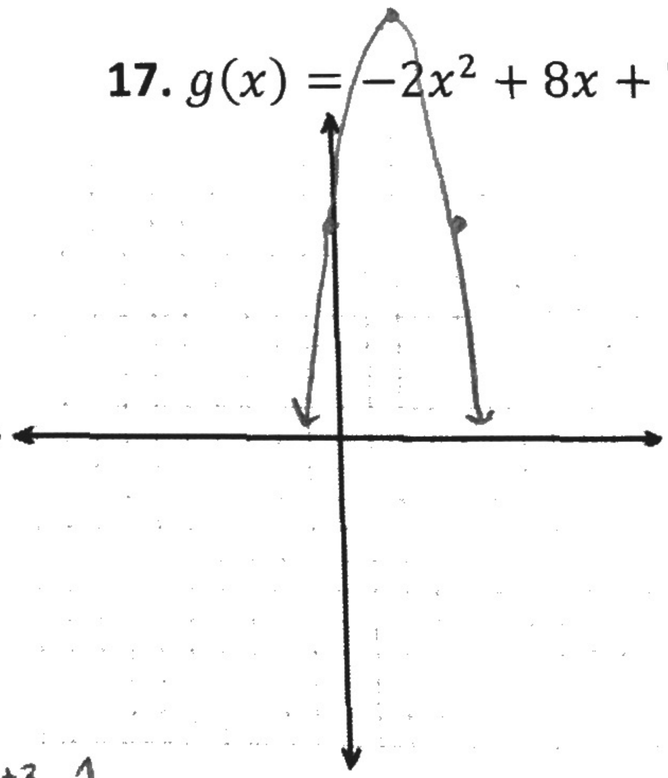
$y = (1+1)(1-3) = -4$

x-int: $p = -1, q = 3$

y-int: $y = (0+1)(0-3)$

$y = -3$

17. $g(x) = -2x^2 + 8x + 7$



aos: $x = \frac{-8}{2(-2)} = 2$ vertex: $(2,15)$

$g(2) = -2(2)^2 + 8(2) + 7 = 15$

y-int: $c = 7$

x-int: $0 = -2x^2 + 8x + 7$

$x = \frac{-8 \pm \sqrt{(8)^2 - 4(-2)(7)}}{2(-2)}$

$x = \frac{-8 \pm 2\sqrt{30}}{-4}$

$x = \frac{-4 \pm \sqrt{30}}{-2}$

Writing Quadratic Functions

18. Write $y = x^2 - 6x + 11$ in vertex form.

$\left(\frac{-6}{2}\right)^2 = 9$

$y - 11 = x^2 - 6x$

$+9 \qquad +9$

$y - 2 = x^2 - 6x + 9$

$y - 2 = (x-3)^2$

$\boxed{y = (x-3)^2 + 2}$

19. Write $f(x) = x^2 - 4x - 1$ in vertex form.

$\left(\frac{-4}{2}\right)^2 = 4$

$f(x) + 1 = x^2 - 4x$

$+4 \qquad +4$

$f(x) + 5 = x^2 - 4x + 4$

$f(x) + 5 = (x-2)^2$

$\boxed{f(x) = (x-2)^2 - 5}$

Write the quadratic function of the parabola in the table in...

x	$f(x)$	
0	24	→ y-intercept $c = 24$
−7	3	
−6	0	→ x-intercept $p = -6$
−5	−1	→ vertex $h = -5, k = -1$
−4	0	→ x-intercept $q = -4$
−3	3	

20. Vertex form

$y = a(x-h)^2 + k$

$y = a(x+5)^2 - 1$

$3 = a(-3+5)^2 - 1$

$3 = 4a - 1$

$4 = 4a$

$1 = a$

$\boxed{y = (x+5)^2 - 1}$

21. Intercept form

$y = a(x-p)(x-q)$

$y = a(x+6)(x+4)$

$-1 = a(-5+6)(-5+4)$

$-1 = a(1)(-1)$

$-1 = -a$

$1 = a$

$\boxed{y = (x+6)(x+4)}$

22. Standard form

$y = ax^2 + bx + c$

$y = ax^2 + bx + 24$

$0 = a(-6)^2 + b(-6) + 24$

$0 = 36a - 6b + 24$

$-24 = 36a - 6b$

$-4 = 6a - b$

$-1(-4 = 6a - b) \rightarrow 4 = -6a + b$

$-1 = a(-5)^2 + b(-5) + 24$

$-1 = 25a - 5b + 24$

$-25 = 25a - 5b$

$-5 = 5a - b$

$-5 = 5(1) - b$

$-5 = 5 + b$

$-10 = -b$

$10 = b$

$4 = -6a + b$

$+ \quad -5 = 5a - b$

$-1 = -a$

$a = 1$

$\boxed{y = x^2 + 10x + 24}$

Solving Quadratic Equations & Imaginary Numbers

Solve the equation using square roots.

23. $(p-4)^2 = 49$

$\sqrt{(p-4)^2} = \sqrt{49}$

$p - 4 = \pm 7$

$p = 4 \pm 7$

$\boxed{p = 11 \text{ or } p = -3}$

24. $4(x-1)^2 + 2 = 10$

$4(x-1)^2 = 8$

$(x-1)^2 = 2$

$\sqrt{(x-1)^2} = \sqrt{2}$

$x - 1 = \pm\sqrt{2}$

$\boxed{x = 1 \pm \sqrt{2}}$

25. $2(x+2)^2 - 5 = 8$

$2(x+2)^2 = 13$

$(x+2)^2 = \frac{13}{2}$

$\sqrt{(x+2)^2} = \sqrt{\frac{13}{2}}$

$x + 2 = \pm\sqrt{\frac{13}{2}}$

$x = -2 \pm \sqrt{\frac{13}{2}}$

$\boxed{x = -2 \pm \frac{\sqrt{26}}{2}}$

Solve the equation by factoring.

26. $0 = x^2 + 6x + 9$

$0 = (x+3)^2$

$0 = x+3$

$\boxed{-3 = x}$

27. $2w^2 - 16w = 12w - 48$

$2w^2 - 28w + 48 = 0$

$w^2 - 14w + 24 = 0$

$(w-12)(w-2) = 0$

$w-12=0 \qquad w-2=0$

$\boxed{w=12} \qquad \boxed{w=2}$

Solve the equation by completing the square.

28. $x^2 + 6x + 3 = 0$ $\left(\frac{6}{2}\right)^2 = 9$

$x^2 + 6x = -3$

$x^2 + 6x + 9 = 6$

$(x+3)^2 = 6$

$x+3 = \pm\sqrt{6}$

$\boxed{x = -3 \pm \sqrt{6}}$

29. $t^2 - 8t - 5 = 0$ $\left(\frac{-8}{2}\right)^2 = 16$

$t^2 - 8t = 5$

$t^2 - 8t + 16 = 21$

$(t-4)^2 = 21$

$t-4 = \pm\sqrt{21}$

$\boxed{t = 4 \pm \sqrt{21}}$

Solve the equation using the Quadratic Formula.

30. $6x^2 - 21x + 1 = 0$ $a=6$ $b=-21$ $c=1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{21 \pm \sqrt{(-21)^2-4(6)(1)}}{2(6)}$$

$$\boxed{x = \frac{21 \pm \sqrt{417}}{12}}$$

31. $2x^2 + 4x = 30$ $2x^2+4x-30=0$ $x^2+2x-15=0$ $a=1$ $b=2$ $c=-15$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2-4(1)(-15)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{64}}{2}$$

$$x = \frac{-2 \pm 8}{2} = -1 \pm 4$$

$x = -1+4 \qquad x = -1-4$

$\boxed{x=3} \qquad \boxed{x=-5}$

$b^2-4ac<0$

2 imaginary solutions

$b^2-4ac=0$

1 real solution

$b^2-4ac>0$

2 real solutions

Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

32. $0 = x^2 - x + 6$ $a=1$ $b=-1$ $c=6$

b^2-4ac

$(-1)^2-4(1)(6)$

$\boxed{-23}<0$

$\boxed{\text{2 imaginary solutions}}$

33. $0 = 4n^2 - 4n - 24$ $a=4$ $b=-4$ $c=-24$

b^2-4ac

$(-4)^2-4(4)(-24)$

$\boxed{400}>0$

$\boxed{\text{2 real solutions}}$

34. $0 = 2x^2 - 20x + 50$ $a=2$ $b=-20$ $c=50$

b^2-4ac

$(-20)^2-4(2)(50)$

$\boxed{0}=0$

$\boxed{\text{one real solution}}$

Simplify.

35. i^{23}

$\frac{23}{4} = 5$ remainder 3

$i^{23} = i^3 = \boxed{-i}$

36. i^{46}

$\frac{46}{4} = 11$ remainder 2

$i^{46} = i^2 = \boxed{-1}$

37. $(12 + 4i) - (3 - 7i)$

$12+4i-3+7i$

$\boxed{9+11i}$

38. $(2 - 15i) - (4 + 5i)$

$2-15i-4-5i$

$\boxed{-2-20i}$

39. $(3 - 2i)(4 + i)$

$12+3i-8i-2i^2$

$12-5i+2(-1)$

$12-5i+2$

$\boxed{14-5i}$

40. $(7 + 5i)(8 - 6i)$

$56-42i+40i-30i^2$

$56-2i+30$

$\boxed{86-2i}$

41. $\frac{5}{1+i} \cdot \frac{(1-i)}{(1-i)}$

$\frac{5-5i}{1-i^2} = \frac{5-5i}{1+1} = \boxed{\frac{5-5i}{2}}$

42. $\frac{5+2i}{3-2i} \cdot \frac{(3+2i)}{(3+2i)}$

$\frac{15+10i+6i+4i^2}{9-4i^2}$

$\frac{15+16i-4}{9+4}$

$\boxed{\frac{11+16i}{13}}$

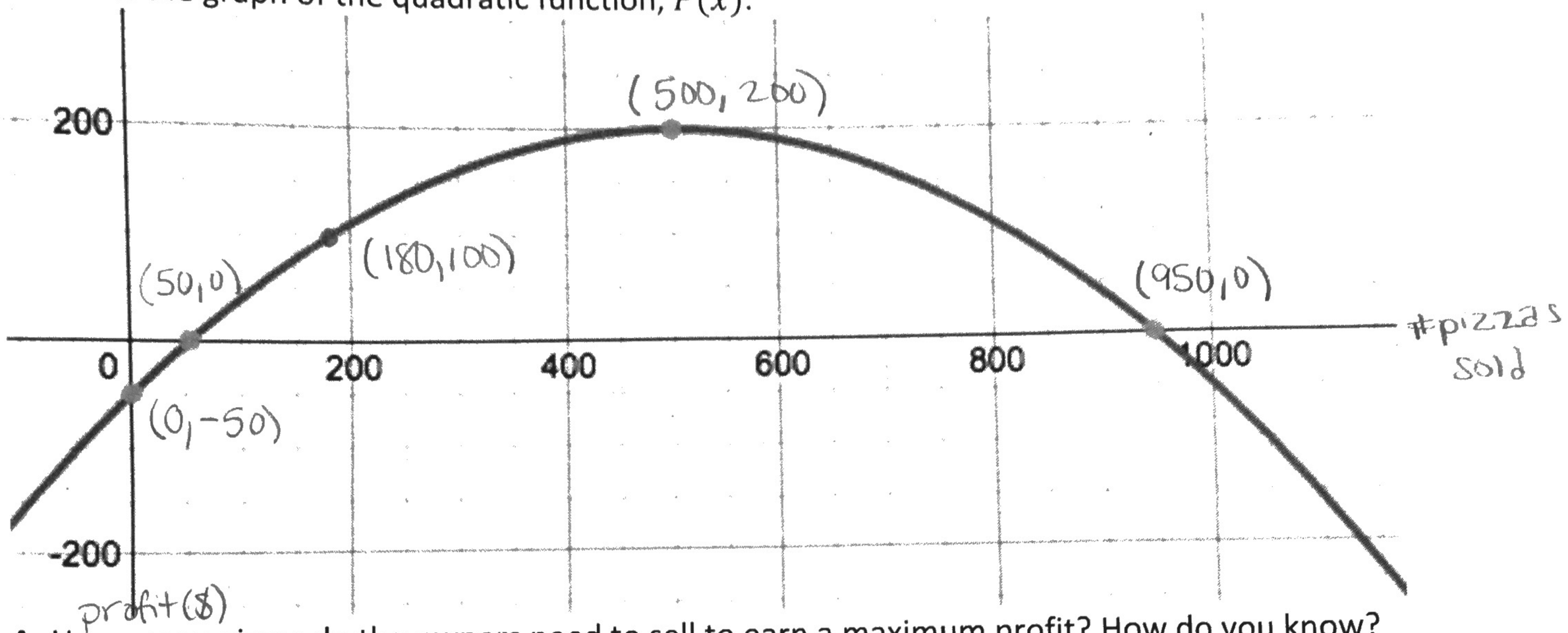
43. $\sqrt{-18}$

$\sqrt{-1}\sqrt{9}\sqrt{2}$

$\boxed{3i\sqrt{2}}$

Quadratic Applications

44. A pizza restaurant sells pizzas (duh). The owners are doing some math (after high school what what) to figure out how many pizzas should be sold to maximize profits. The graph below represents the profit, $P(x)$, in dollars, earned by selling x pizzas. Answer the questions below based on the graph of the quadratic function, $P(x)$.



A. How many pizzas do the owners need to sell to earn a maximum profit? How do you know?

500 pizzas because that is the x-value of the vertex.

B. What is $P(0)$? What does this value mean in terms of the real-life problem? Why does this sometimes happen in real life?

P(0) = -50. P(0) is the initial profit. Negative profit is start-up costs.

(i.e. you need to spend $ to make $)

C. If the pizza restaurant wants to make a profit of about $100, approximately how many pizzas should be sold?

Approximately 180 pizzas.

D. What is the minimum number of pizza that must be sold in order for the restaurant to have a *positive* profit?

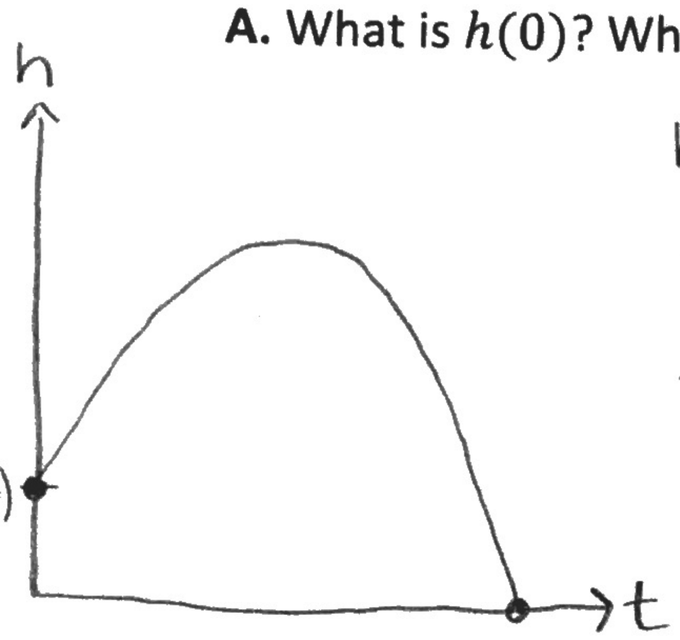
51 pizzas.

E. What is the maximum profit? How do you know?

$200 because that is the y-value of the vertex.

45. The function $h(t) = -16t^2 + 40t + 4$ models the height, $h(t)$, in feet of a ball t seconds after being tossed into air by a juggler.

A. What is $h(0)$? What does $h(0)$ represent in real-life in this word problem?



$h(0) = 4$

$h(0) = 4$ ft represents the initial height the juggler tosses the ball in the air.

B. What is the maximum height of the ball? How do you know?

aos: $x = \frac{-b}{2a}$

$x = \frac{-40}{2(-16)} = 1.25$

Vertex: $(1.25, 29)$

$h(1.25) = -16(1.25)^2 + 40(1.25) + 4 = 29$

29 feet because it is the y-value of the vertex.

C. How long is the ball in the air? How do you know?

$0 = -16t^2 + 40t + 4$

$0 = -4t^2 + 10t + 1$

$t = \frac{-10 \pm \sqrt{(10)^2 - 4(-4)(1)}}{2(-4)}$

$t = \frac{-10 \pm \sqrt{116}}{-8}$

$t = \frac{-10 \pm 2\sqrt{29}}{-8}$

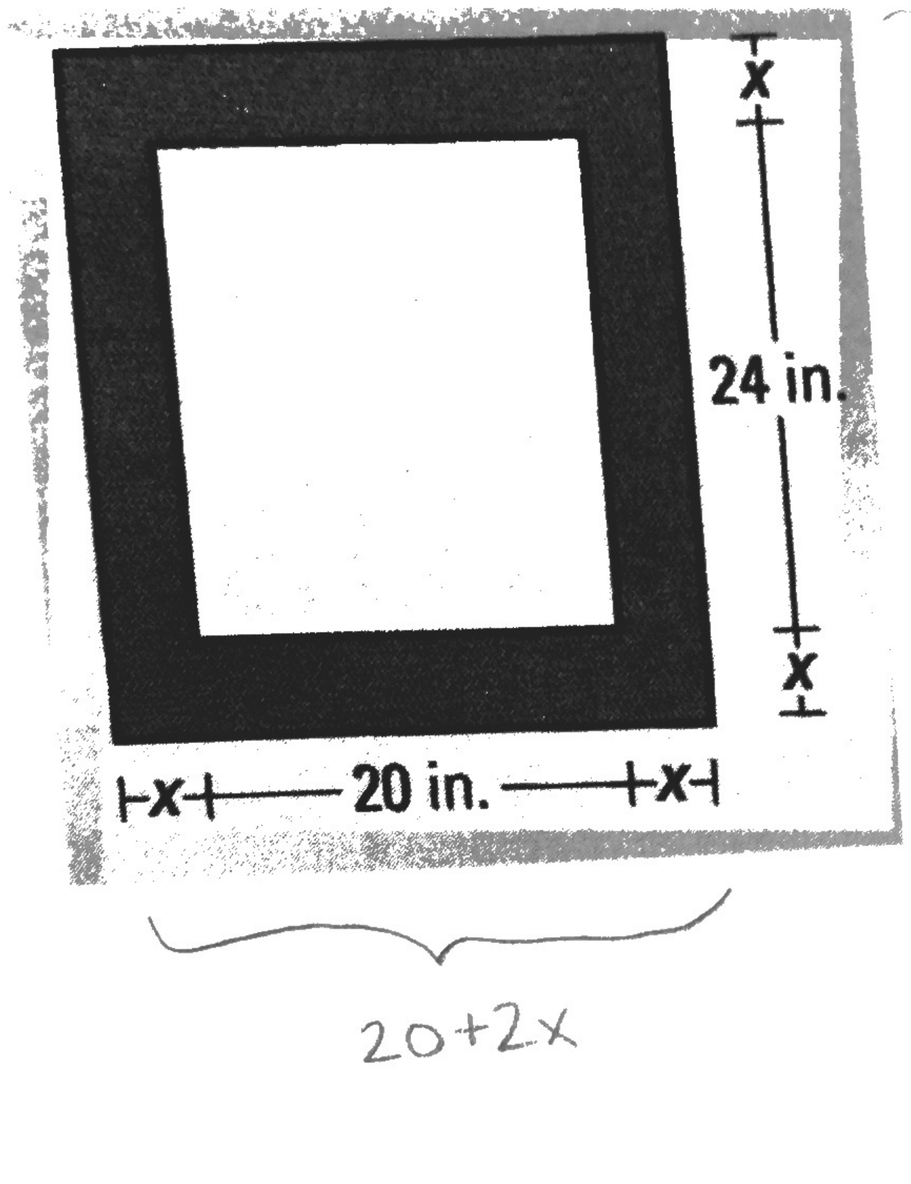
$t = \frac{-5 \pm \sqrt{29}}{-4}$

$t = \frac{-5 + \sqrt{29}}{-4}$ $\quad t = \frac{-5 - \sqrt{29}}{-4}$

$t = -0.096$ $\quad t = 2.596$

2.596s because it is the positive x-intercept.

46. You are creating a metal border of uniform width for a rectangular wall mirror that is 20 inches by 24 inches. You have 416 square inches of metal to use. What is the greatest possible width x of the border?



20+2x

24+2x

area of border = area of border + mirror − area of mirror

$416 = (20+2x)(24+2x) - (20)(24)$

$416 = 480 + 40x + 48x + 4x^2 - 480$

$416 = 4x^2 + 88x$

$0 = 4x^2 + 88x - 416$

$0 = x^2 + 22x - 104$

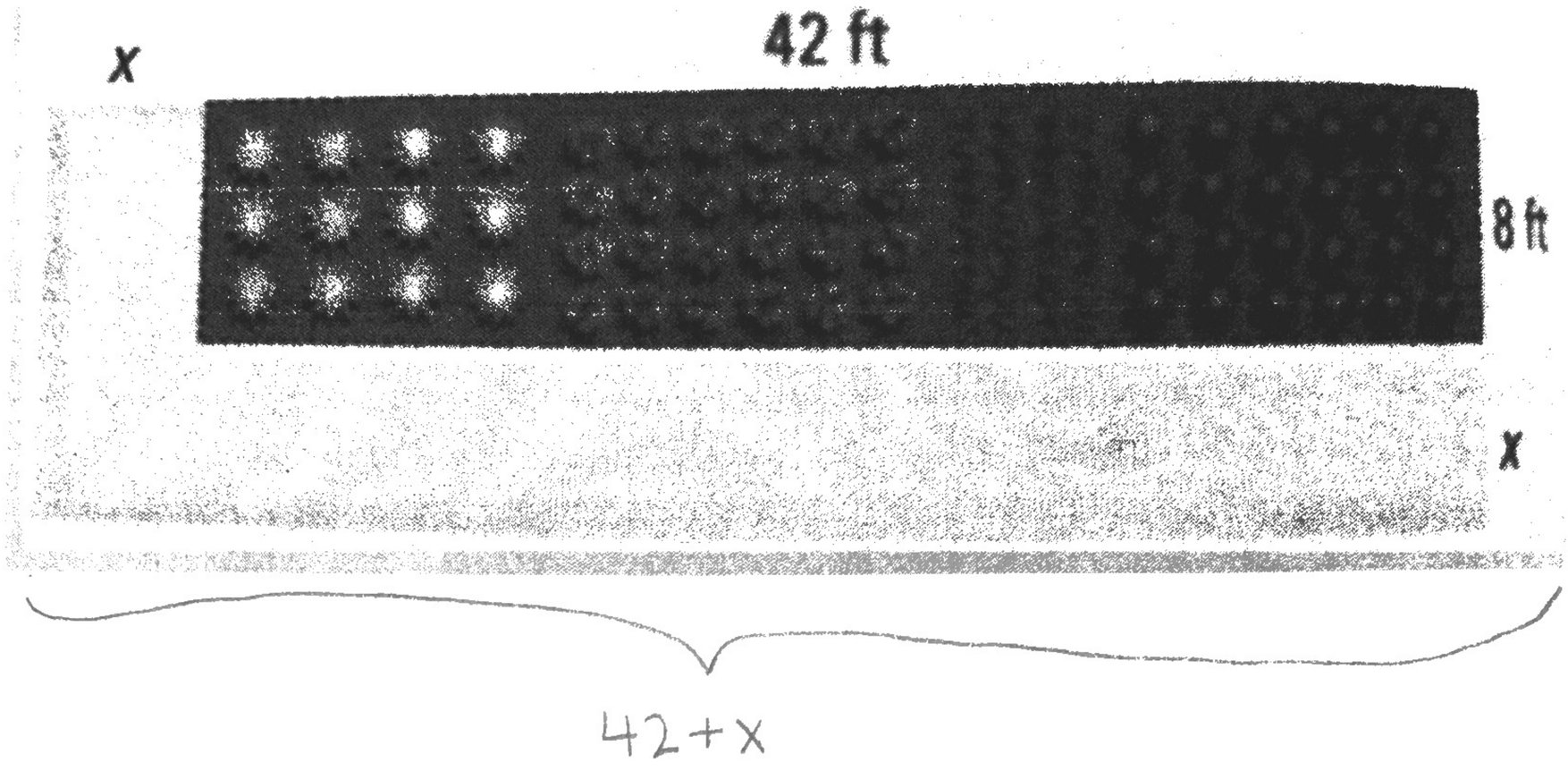
$0 = (x+26)(x-4)$

$x+26=0$ $\quad$ $x-4=0$

$x=-26$ $\quad$ $\boxed{x=4}$

$\boxed{4 \text{ inches}}$

47. You have a rectangular vegetable garden that measures 42 feet by 8 feet. You want to double the area of the park by expanding the length and width as shown. What is the value of x? What are the dimensions of the new garden?



new area = new length × new width

$2(42)(8) = (42+x)(8+x)$

$672 = 336 + 42x + 8x + x^2$

$672 = x^2 + 50x + 336$

$0 = x^2 + 50x - 336$

$0 = (x+56)(x-6)$

$0 = x+56$ $\quad$ $0 = x-6$

$x = -56$ $\quad$ $\boxed{x = 6 \text{ ft}}$

$8+6 = \boxed{14 \text{ ft}}$

$42+6 = \boxed{48 \text{ ft}}$