

Quadratic Applications Study Guide

Solving Nonlinear Systems

1. Solve the system by elimination.

$$\begin{array}{r} 2x^2 - 8x + y = -5 \\ + 2x^2 - 16x - y = -31 \\ \hline 4x^2 - 24x = -36 \\ 4x^2 - 24x + 36 = 0 \\ x^2 - 6x + 9 = 0 \\ (x-3)^2 = 0 \\ x-3 = 0 \\ x = 3 \end{array}$$

$$\begin{array}{r} 2(3)^2 - 8(3) + y = -5 \\ 2(9) - 24 + y = -5 \\ 18 - 24 + y = -5 \\ -6 + y = -5 \\ y = 1 \end{array}$$

(3, 1)

Solve the system algebraically using substitution or elimination.

2. $2x^2 - 2 = y$
 $-2x + 2 = y$

$$2x^2 - 2 = -2x + 2$$

$$2x^2 + 2x - 4 = 0$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x+2=0 \quad x-1=0$$

$$x=-2 \quad x=1$$

$$y = -2(-2) + 2$$

$$y = 4 + 2$$

$$y = 6$$

(-2, 6)

$$y = -2(1) + 2$$

$$y = -2 + 2$$

$$y = 0$$

(1, 0)

3. $x^2 - 6x + 13 = y$
 $-y = -2x + 3$

$$x^2 - 6x + 13 = y$$

$$+ \quad \frac{-2x + 3 = -y}{\hline}$$

$$x^2 - 8x + 16 = 0$$

$$(x-4)^2 = 0$$

$$x-4=0$$

$$x=4$$

$$y = (4)^2 - 6(4) + 13$$

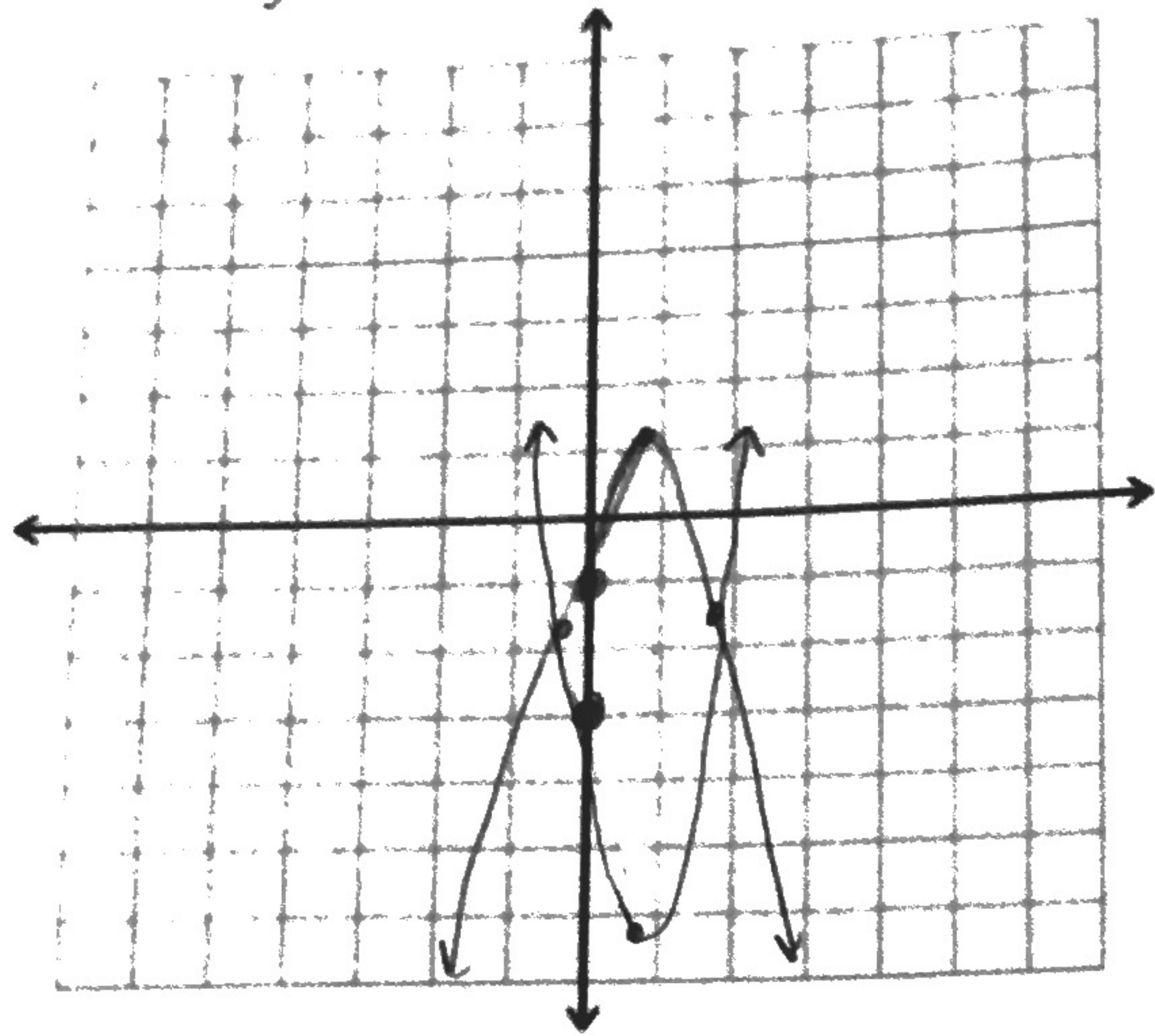
$$y = 16 - 24 + 13$$

$$y = -8 + 13$$

$$y = 5$$

(4, 5)

4. Solve $y = -3x^2 + 5x - 1$
 $y = 5x^2 - 8x - 3$ by graphing.



$$y = -3x^2 + 5x - 1$$

$$x = \frac{-5}{2(-3)} = \frac{5}{6}$$

$$y = -3\left(\frac{5}{6}\right)^2 + 5\left(\frac{5}{6}\right) - 1$$

$$y = 1.08\bar{3}$$

$$y = 5x^2 - 8x - 3$$

$$x = \frac{-8}{2(5)} = \frac{8}{10} = \frac{4}{5}$$

$$y = 5\left(\frac{4}{5}\right)^2 - 8\left(\frac{4}{5}\right) - 3$$

$$y = -6.2$$

$$(-0.142, -1.768)$$

$$(1.767, -1.529)$$

Word Problems

Launched Object

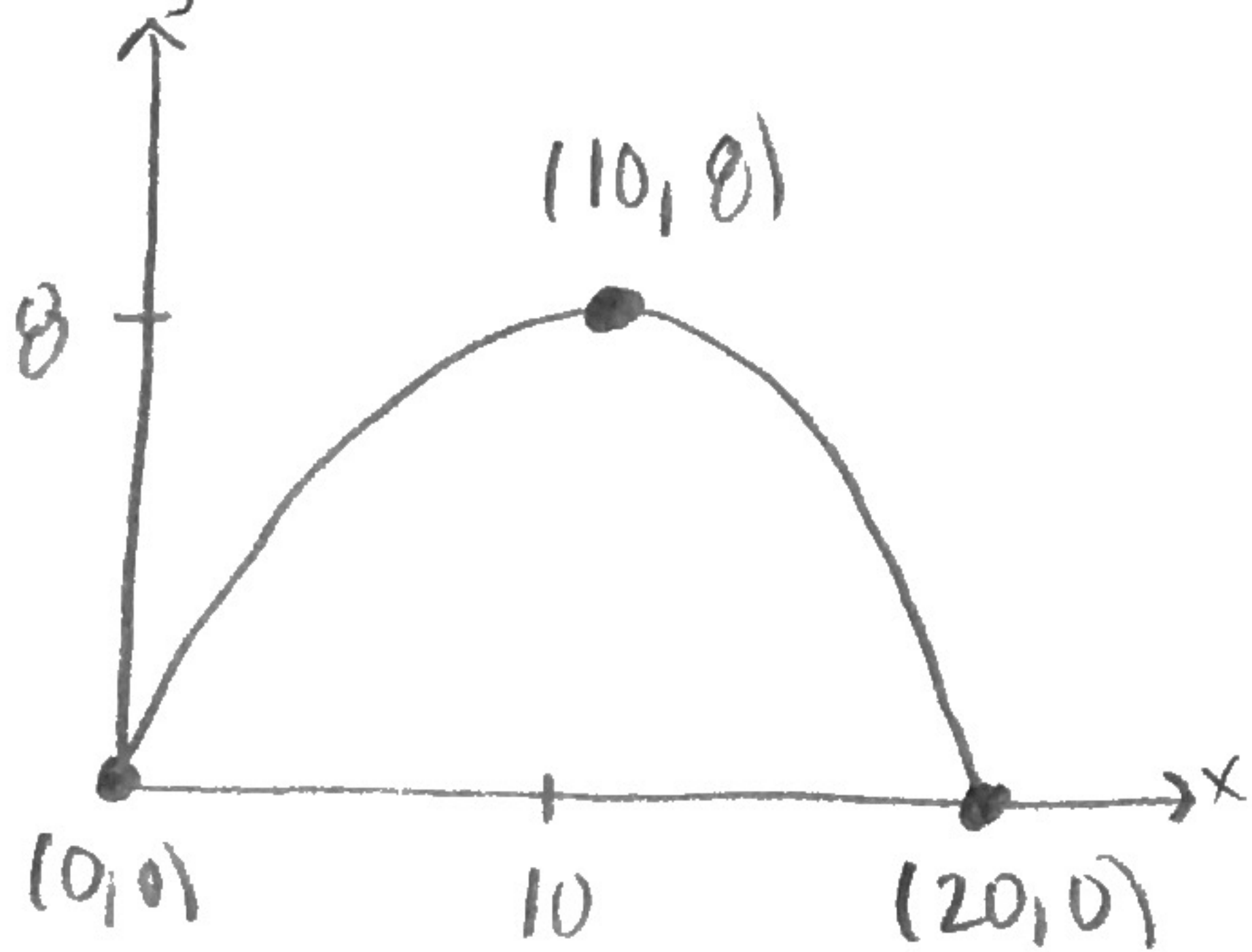
5. A soccer player kicks a ball downfield. The height of the ball increases until it reaches a maximum height of 8 yards, 20 yards away from the player. A second kick is modeled by $y = x(0.4 - 0.008x)$. Which kick travels farther before hitting the ground? Which kick travels higher?



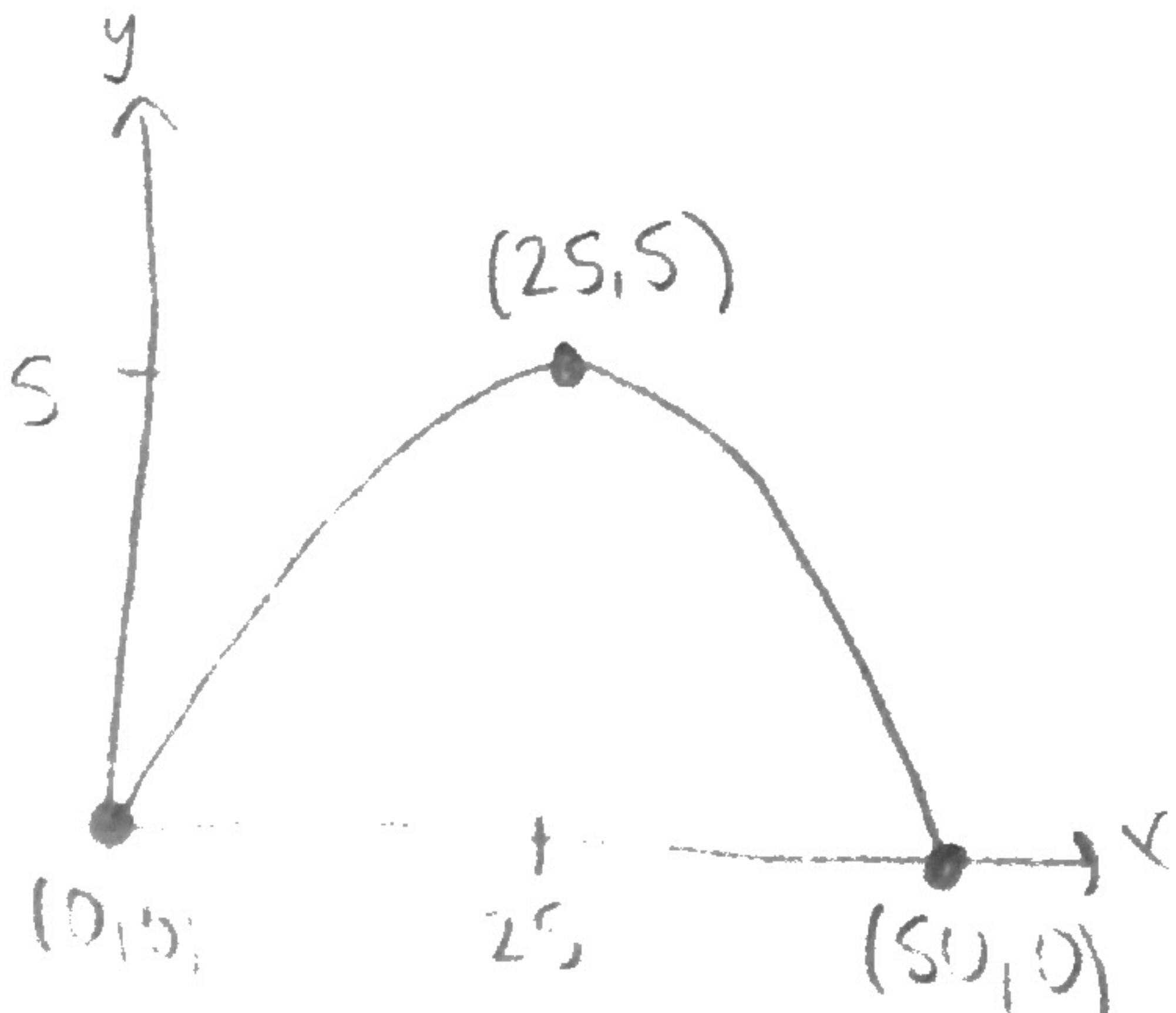
The second kick travels farther.

The first kick travels higher.

First kick



Second kick



$$y = x(0.4 - 0.008x)$$

$$0 = x(0.4 - 0.008x)$$

$$x = 0 \quad 0.4 - 0.008x = 0$$

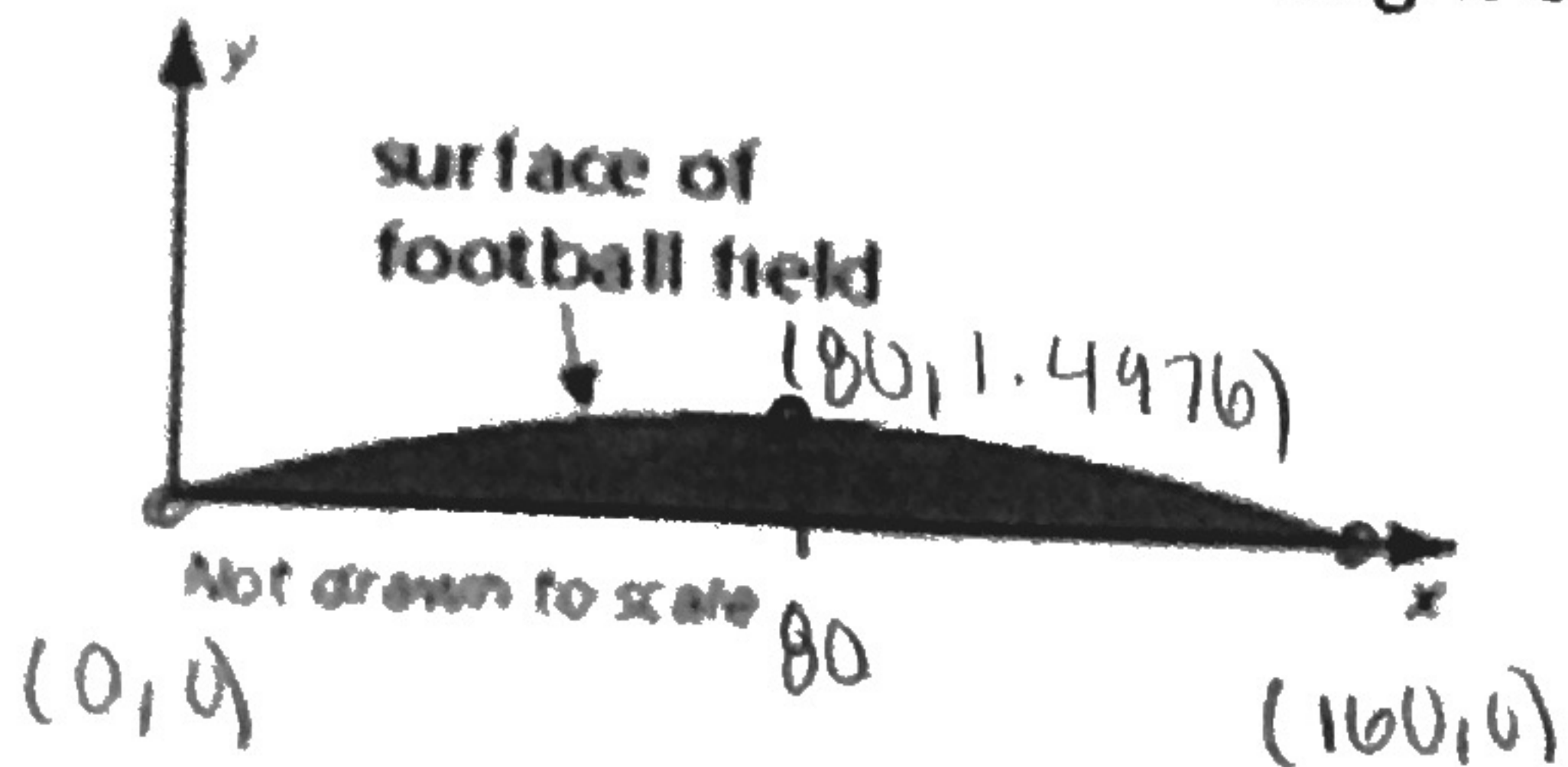
$$0.4 = 0.008x$$

$$50 = x$$

$$y = 25(0.4 - 0.008 \cdot 25)$$

$$y = 5$$

6. Although a football field appears to be flat, some are actually shaped like a parabola so that rain runs off to both sides. The cross section of a field can be modeled by $y = -0.000234x(x - 160)$, where x and y are measured in feet. What is the width of the field? What is the maximum height of the surface of the field?



$$0 = -0.000234x(x - 160)$$

$$-0.000234x = 0 \quad x - 160 = 0$$

$$x = 0 \quad x = 160$$

$$\boxed{\text{width of field} = 160 \text{ ft}}$$

$$x = 80$$

$$y = -0.000234(80)(80 - 160)$$

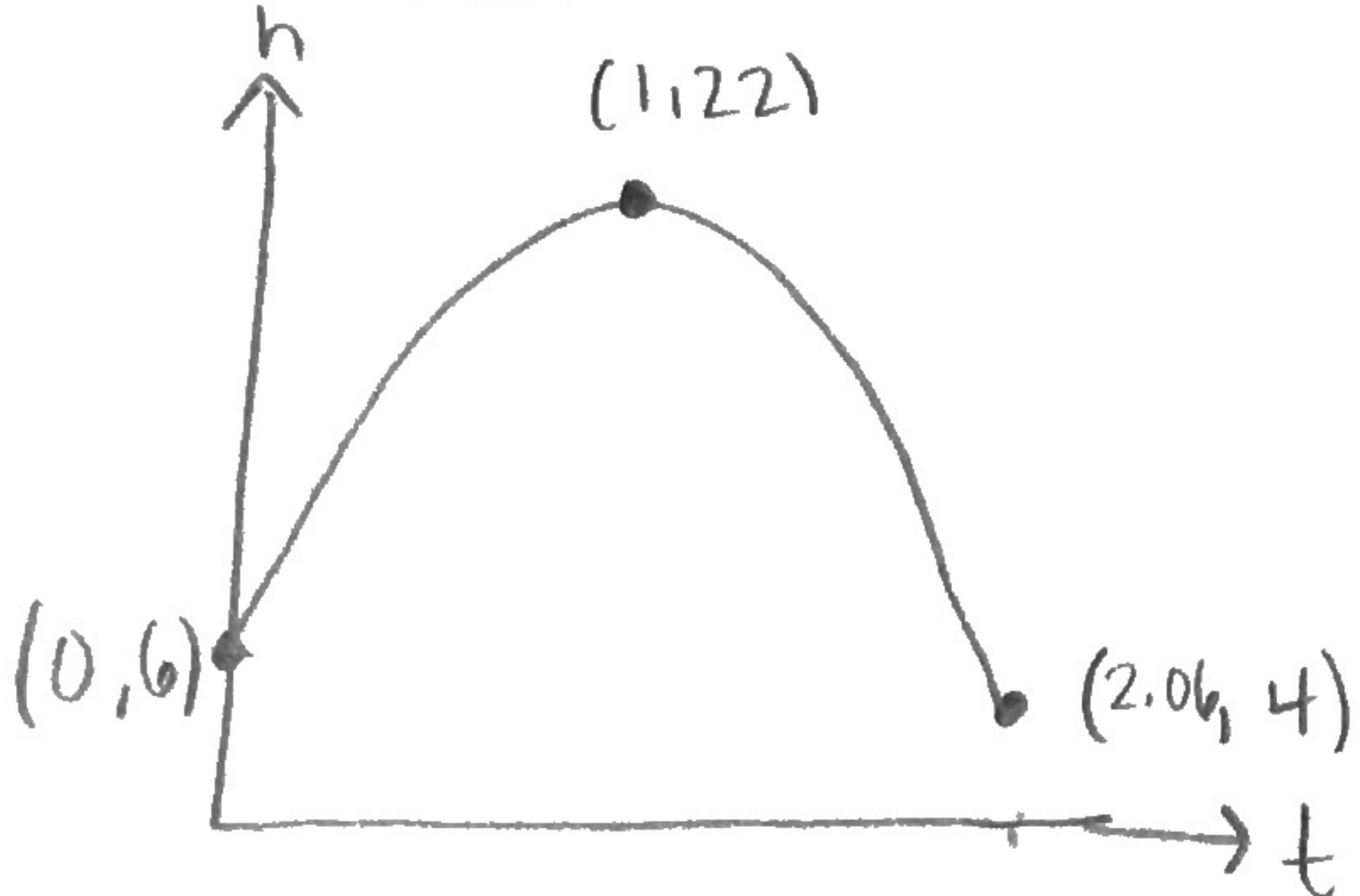
$$y = 1.4976$$

$$\boxed{\text{max height} = 1.4976 \text{ ft}}$$

7. While marching, a drum major tosses a baton into the air and catches it. The height h (in feet) of the baton t seconds after it is thrown can be modeled by the function $h = -16t^2 + 32t + 6$.

A. Find the maximum height of the baton.

B. The drum major catches the baton when it is 4 feet above the ground. How long is the baton in the air?



$$\textcircled{A} \quad t = \frac{-32}{2(-16)} = 1$$

$$\boxed{\text{max height} = 22 \text{ ft}}$$

$$h = -16(1)^2 + 32(1) + 6$$

$$h = 22$$

$$\textcircled{B} \quad 4 = -16t^2 + 32t + 6$$

$$0 = -16t^2 + 32t + 2$$

$$0 = -8t^2 + 16t + 1$$

$$t = \frac{-16 \pm \sqrt{(16)^2 - 4(-8)(1)}}{2(-8)}$$

$$t = \frac{-4 + 3\sqrt{2}}{-4} \approx -0.01$$

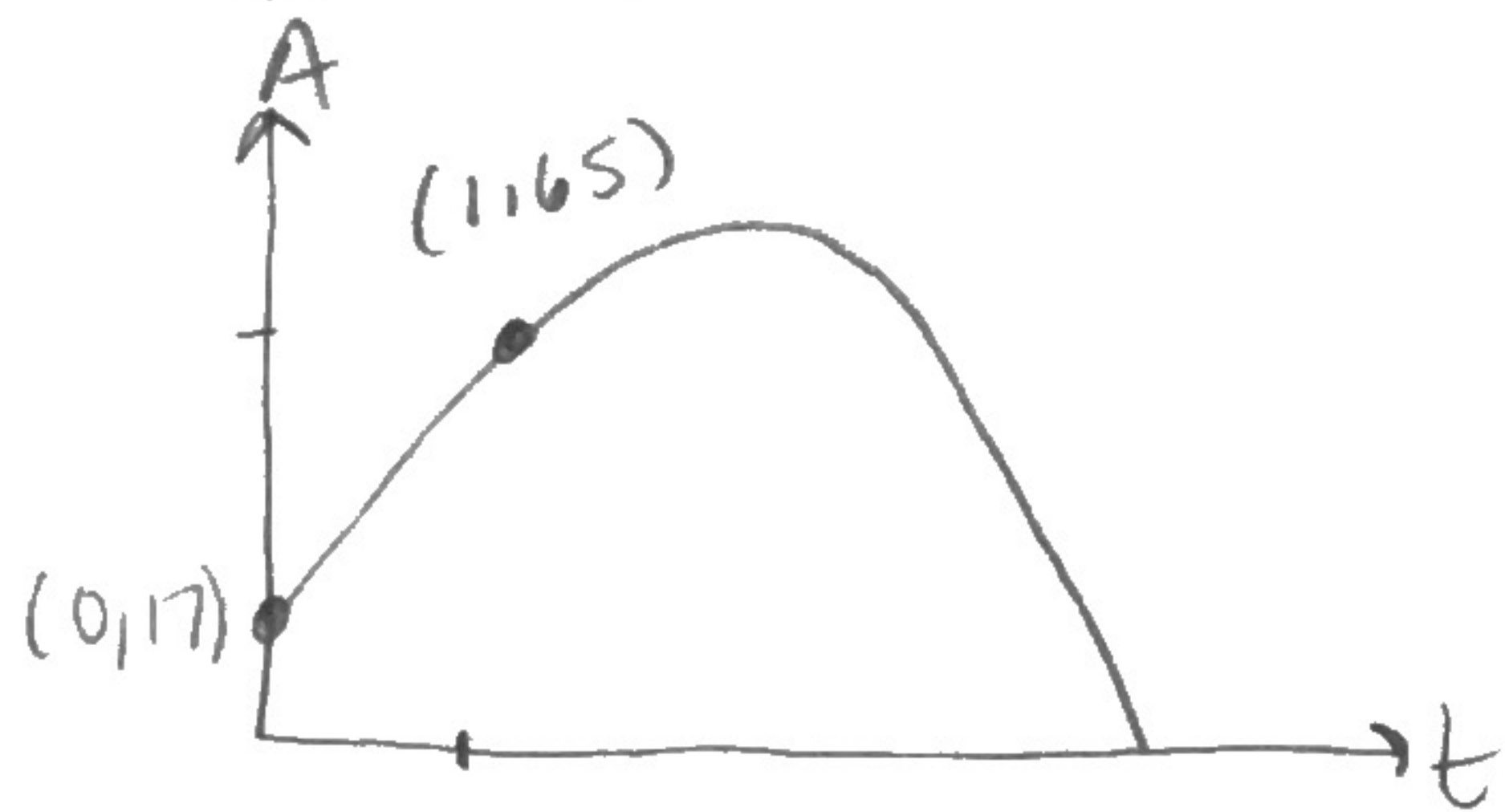
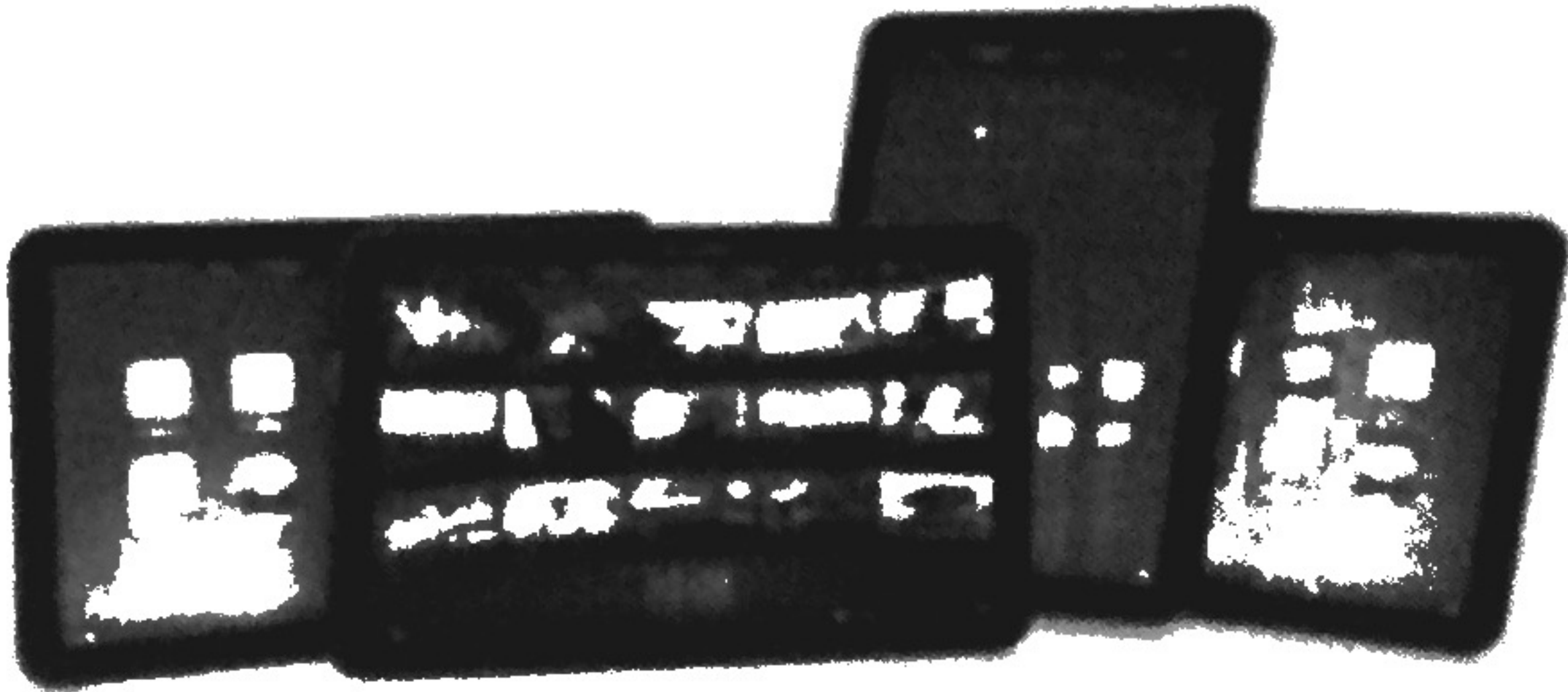
$$t = \frac{-4 - 3\sqrt{2}}{-4} \approx 2.06$$

$$t = \frac{-16 \pm 12\sqrt{2}}{-16}$$

$$\boxed{\text{About } 2.06 \text{ s}}$$

$$t = \frac{-4 \pm 3\sqrt{2}}{-4}$$

8. A number A of tablet computers sold (in millions) can be modeled by the function $A = 4.5t^2 + 43.5t + 17$, where t represents the year after 2010. In what year did the tablet computer sales reach 65 million?



2011

$$A = 65$$

$$65 = 4.5t^2 + 43.5t + 17$$

$$0 = 4.5t^2 + 43.5t - 48$$

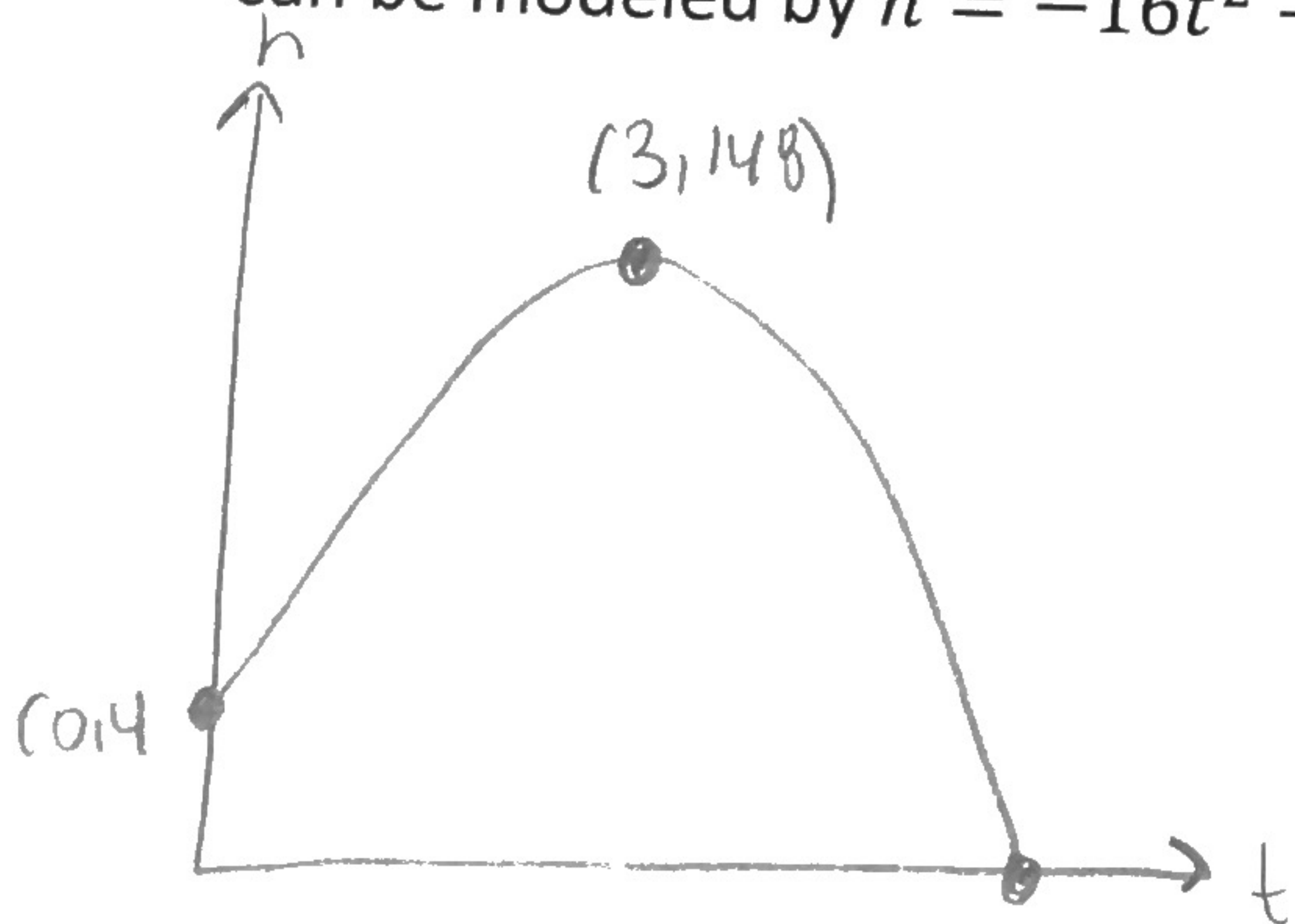
$$t = \frac{-43.5 \pm \sqrt{(43.5)^2 - 4(4.5)(-48)}}{2(4.5)}$$

$$t = \frac{-43.5 \pm 52.5}{9}$$

$$x = \frac{-43.5 + 52.5}{9} = 1$$

$$x = \frac{-43.5 - 52.5}{9} = -10\frac{2}{3}$$

9. An employee at a local stadium is launching T-shirts from a T-shirt cannon into the crowd during an intermission of a football game. The height h (in feet) of the T-shirt after t seconds can be modeled by $h = -16t^2 + 96t + 4$. Find the maximum height of the T-shirt.



$$t = \frac{-96}{2(-16)} = 3$$

$$h = -16(3)^2 + 96(3) + 4$$

$$h = 148$$

max height = 148 ft

Maximizing Revenue

9. A skateboard shop sells 50 skateboards per week when the advertised price is charged. For each \$1 decrease in price, one additional skateboard per week is sold. The shop's revenue can be modeled by $y = (70 - x)(50 + x)$. Find the maximum weekly revenue.

SKATEBOARDS

Quality
Skateboards
for \$70

$x = \#$ price decrease $y =$ weekly revenue

$$y = (70 - x)(50 + x)$$

$$0 = (70 - x)(50 + x)$$

$$70 - x = 0 \quad 50 + x = 0$$

$$x = 70 \quad x = -50$$

$$x = \frac{70 + (-50)}{2} = 10$$

$$y = (70 - 10)(50 + 10)$$

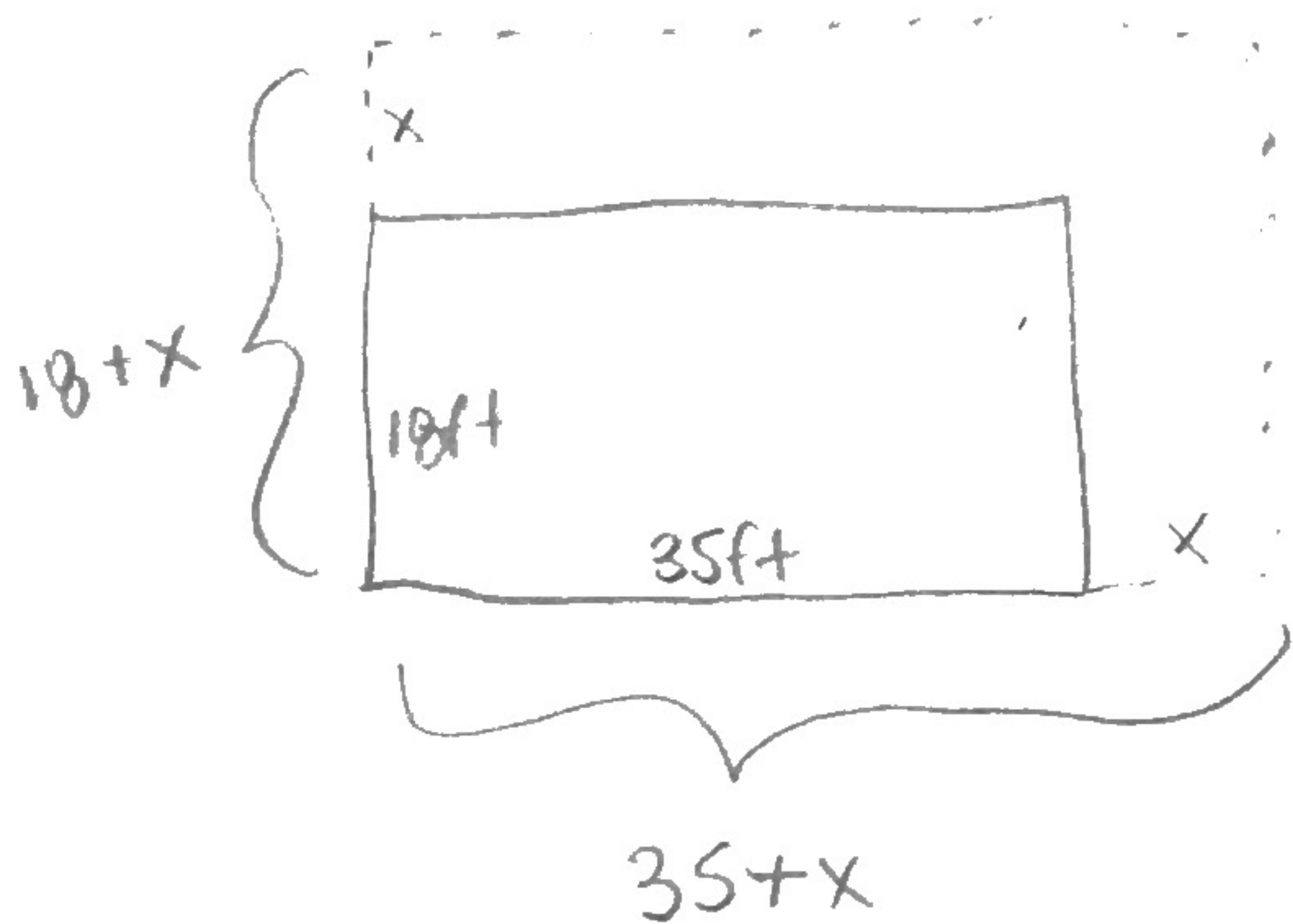
$$y = 3600$$

$$\text{charge} = 70 - 10 = 60$$

$$\text{max revenue} = \$3600$$

Border/Area Problems

10. A rectangular enclosure at the zoo is 35 feet long by 18 feet wide. The zoo wants to double the area of the enclosure by adding the same distance x to the length and width. Write and solve an equation to find the value of x . What are the dimensions of the enclosure?



new area = new length \cdot new width

$$2(35)(18) = (18+x)(35+x)$$

$$1260 = 630 + 53x + x^2$$

$$0 = x^2 + 53x - 630$$

$$0 = (x + 63)(x - 10)$$

$$x + 63 = 0$$

$$x - 10 = 0$$

$$x = -63$$

$$x = 10$$

$$\text{new length} = 35 + 10 = 45 \text{ft}$$

$$\text{new width} = 18 + 10 = 28 \text{ft}$$