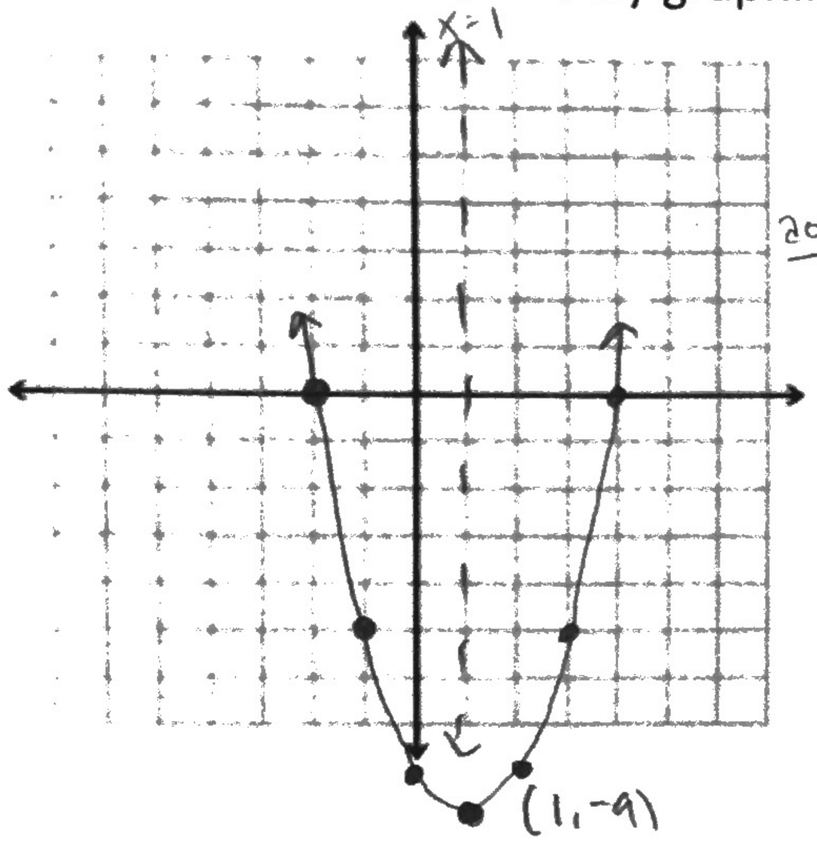
Name: Key Date: Band:
Algebra 2

Unit 3: Quadratic Equations and Complex Numbers Study Guide

3.1 Solving Quadratic Equations

1. Solve $x^2 - 2x - 8 = 0$ by graphing.

$x = -2, x = 4$



$y = x^2 - 2x - 8$

y-int: $(0, -8)$

axis: $x = \frac{-b}{2a} = \frac{2}{2(1)} = 1$

vertex: $(1, -9)$

$y = (1)^2 - 2(1) - 8$

$y = -9$

x	y
-2	0
-1	-5
0	-8
1	-9
2	-8
3	-5
4	0

Solve the equation using square roots or by factoring.

2. $3x^2 - 4 = 8$

$+4 \quad +4$

$\frac{3x^2}{3} = \frac{12}{3}$

$x^2 = 4$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

3. $x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$

$x + 8 = 0 \quad x - 2 = 0$

$-8 \quad -8 \quad +2 \quad +2$

$x = -8 \quad x = 2$

4. $2x^2 - 17x = -30$

$+30 \quad +30$

$2x^2 - 17x + 30 = 0$

$(2x - 5)(x - 6) = 0$

$2x - 5 = 0 \quad x - 6 = 0$

$+5 \quad +5 \quad +6 \quad +6$

$\frac{2x}{2} = \frac{5}{2} \quad x = 6$

$x = 5/2$

3.2 Complex Numbers

Perform each operation. Write the answer in standard form. $a + bi$

5. $(3 - 6i) - (7 + 2i)$

$3 - 6i - 7 + 2i$

$-4 - 8i$

6. $5i(4 + 5i)$

$5i(4) + 5i(5i)$

$20i + 25i^2$

$20i + 25(-1)$

$20i - 25$

$-25 + 20i$

7. Find the values of x and y that satisfy the equation $36 - yi = 4x + 3i$.

real

$36 = 4x$

$\frac{4}{4} \quad \frac{4}{4}$

$9 = x$

imaginary

$-yi = 3i$

$\frac{-i}{-i} \quad \frac{-i}{-i}$

$y = -3$

Perform the operation. Write the answer in standard form. $a+bi$

8. $(-2+3i)+(7-6i)$

$-2+3i+7-6i$

$5-3i$

9. $(9+3i)-(-2-7i)$

$9+3i+2+7i$

$11+10i$

10. $(5+6i)(-4+7i)$

$5(-4)+5(7i)+6i(-4)+6i(7i)$

$-20+35i-24i+42i^2$

$-20+11i+42(-1)$

$-20+11i-42$

$-62+11i$

11. Solve $7x^2+21=0$. $i=\sqrt{-1}$

$-21\quad -21$

$\frac{7x^2}{7}=\frac{-21}{7}$

$x^2=-3$

$\sqrt{x^2}=\sqrt{-3}$

$x=\pm\sqrt{-3}$

$x=\pm\sqrt{-1}\cdot\sqrt{3}$

$x=\pm i\sqrt{3}$

12. Find the zeros of $f(x)=2x^2+32$.

$0=2x^2+32$

$-32\quad -32$

$\frac{-32}{2}=\frac{2x^2}{2}$

$-16=x^2$

$\sqrt{-16}=\sqrt{x^2}$

$\pm\sqrt{-16}=x$

$x=\pm\sqrt{-1}\cdot\sqrt{16}$

$x=\pm i\cdot 4$

$x=\pm 4i$

3.3 Completing the Square

13. Solve $x^2+12x+8=0$ by completing the square.

$-8\quad -8$

$x^2+12x=-8$

$+36\quad +36$

$x^2+12x+36=28$

$(x+6)^2=28$

$b=12$ $\left(\frac{b}{2}\right)^2=\left(\frac{12}{2}\right)^2=6^2=36$

$x+6=\pm\sqrt{28}$

$-6\quad -6$

$x=-6\pm\sqrt{28}$

$x=-6\pm 2\sqrt{7}$

Solve the equation by completing the square.

64 **14.** $x^2+16x+17=0$

$-17\quad -17$

$x^2+16x=-17$

$+64\quad +64$

$x^2+16x+64=47$

$(x+8)^2=47$

$x+8=\pm\sqrt{47}$

$x=-8\pm\sqrt{47}$

15. $\frac{4x^2+16x+25=0}{4}$ $\left(\frac{4}{2}\right)^2=4$

$x^2+4x+\frac{25}{4}=0$

$-\frac{25}{4}\quad -\frac{25}{4}$

$x^2+4x=-\frac{25}{4}$

$+4\quad +4$

$x^2+4x+4=-\frac{9}{4}$

$(x+2)^2=-\frac{9}{4}$

$x+2=\pm\sqrt{-9/4}$

$x=-2\pm\sqrt{-9/4}$

$x=-2\pm\frac{3}{2}i$

16. $\frac{9x(x-6)}{9}=\frac{81}{9}$ $\left(\frac{-6}{2}\right)^2=9$

$x(x-6)=9$

$x^2-6x=9$

$+9\quad +9$

$x^2-6x+9=18$

$(x-3)^2=18$

$x-3=\pm\sqrt{18}$

$+3\quad +3$

$x=3\pm\sqrt{18}$ 2

$x=3\pm 3\sqrt{2}$

17. Write $y = x^2 - 2x + 20$ in vertex form. Then identify the vertex.

$y + ? = (x^2 - 2x + ?) + 20$ $\quad \left(\frac{-2}{2}\right)^2 = 1$

$y + 1 = (x^2 - 2x + 1) + 20$

$y + 1 = (x-1)^2 + 20$

$\boxed{y = (x-1)^2 + 19}$ $\quad \boxed{\text{vertex: } (1, 19)}$

3.4 Using the Quadratic Formula

18. Solve $-x^2 + 4x = 5$ using the Quadratic Formula.

$-x^2 + 4x - 5 = 0$ $\quad a = -1,\ b = 4,\ c = -5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-4 \pm \sqrt{16 - 20}}{-2}$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(-1)(-5)}}{2(-1)}$

$x = \frac{-4 \pm \sqrt{-4}}{-2}$

$x = -4 \pm 2i$

$x = \frac{-4 + 2i}{-2}$ $\quad x = \frac{-4 - 2i}{-2}$

$\boxed{x = 2 - i}$ $\quad \boxed{x = 2 + i}$

Solve the equation using the Quadratic Formula.

19. $-x^2 + 5x = 2$

$-x^2 + 5x - 2 = 0$

$a = -1,\ b = 5,\ c = -2$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(-1)(-2)}}{2(-1)}$

$x = \frac{-5 \pm \sqrt{25 - 8}}{-2}$

$x = \frac{-5 \pm \sqrt{17}}{-2}$

$\boxed{x = \frac{-5 + \sqrt{17}}{-2}}$ $\quad \boxed{x = \frac{-5 - \sqrt{17}}{-2}}$

20. $2x^2 + 5x = 3$

$2x^2 + 5x - 3 = 0$

$a = 2,\ b = 5,\ c = -3$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-3)}}{2(2)}$

$x = \frac{-5 \pm \sqrt{25 + 24}}{4}$

$x = \frac{-5 \pm \sqrt{49}}{4}$ $\quad x = \frac{-5 \pm 7}{4}$

$x = \frac{-5 + 7}{4}$ $\quad x = \frac{-5 - 7}{4}$

$x = \frac{2}{4}$ $\quad x = \frac{-12}{4}$

$\boxed{x = \frac{1}{2}}$ $\quad \boxed{x = -3}$

21. $3x^2 - 12x + 13 = 0$

$a = 3,\ b = -12,\ c = 13$

$x = \frac{12 \pm \sqrt{(-12)^2 - 4(3)(13)}}{2(3)}$

$x = \frac{12 \pm \sqrt{144 - 156}}{6}$

$x = \frac{12 \pm \sqrt{-12}}{6}$

$x = \frac{12 \pm 2i\sqrt{3}}{6}$

$x = \frac{12 + 2i\sqrt{3}}{6}$ $\quad x = \frac{12 - 2i\sqrt{3}}{6}$

$\boxed{x = \frac{6 + i\sqrt{3}}{3}}$ $\quad \boxed{x = \frac{6 - i\sqrt{3}}{3}}$

b^2-4ac

Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

22. $-x^2 - 6x - 9 = 0$

$a=-1 \quad b=-6 \quad c=-9$

$(-6)^2-4(-1)(-9)$

$36-36 = \boxed{0}$

$\boxed{\text{One real solution}}$

23. $x^2 - 2x - 9 = 0$

$a=1 \quad b=-2 \quad c=-9$

$(-2)^2-4(1)(-9)$

$4+36 = \boxed{40} > 0$

$\boxed{\text{two real solutions}}$

24. $x^2 + 6x + 5 = 0$

$a=1 \quad b=6 \quad c=5$

$(6)^2-4(1)(5)$

$36-20 = \boxed{16} > 0$

$\boxed{\text{two real solutions}}$

★ if $b^2-4ac < 0$ then two imaginary solutions ★