Name: Key Date:
Geometry Band:

Reasoning and Proof Study Guide

2.1 Conditional Statements

1. Write the if-then form, the converse, the inverse, the contrapositive, and the biconditional of the conditional statement "A leap year is a year with 366 days."

if-then: If a year is a leap year, then it is a year with 366 days.

converse: If a year has 366 days, then it is a leap year.

inverse: If a year is not a leap year, then it is not a year with 366 days.

contrapositive: If it is not a year with 366 days, then it is not a leap year.

biconditional: A year is a leap year if and only if it has 366 days.

Write the if-then form, the converse, the inverse, the contrapositive, and the biconditional of the conditional statement.

2. Two lines intersect in a point.

if-then: If two lines intersect, then they intersect at a point.

converse: If two lines intersect at a point, then they intersect.

inverse: If two lines do not intersect, then they do not intersect at a point

contrapositive: If two lines do not intersect at a point, then they do not intersect.

biconditional: Two lines intersect if and only if they intersect at a point

3. $4x + 9 = 21$ because $x = 3$.

if-then: If $x=3$, then $4x+9=21$.

converse: If $4x+9=21$, then $x=3$.

inverse: If $x\neq3$, then $4x+9\neq21$.

contrapositive: If $4x+9\neq21$, then $x\neq3$.

biconditional: $x=3$ if and only if $4x+9=21$.

4. Supplementary angles sum to 180°.

if-then: If angles are supplementary, then they sum to 180°.

converse: If angles sum to 180°, then angles are supplementary.

inverse: If angles are not supplementary, then they do not sum to 180°.

contrapositive: If angles do not sum to 180°, then they are not supplementary.

biconditional: Angles are supplementary if and only if they sum to 180°.

5. Right angles are 90°.

if-then: If angles are right angles, then they equal 90°.

converse: If angles equal 90°, then they are right angles.

inverse: If angles are not right angles, then they do not equal 90°.

contrapositive: If angles do not equal 90°, then they are not right angles.

biconditional: Angles are right angles if and only if they equal 90°.

2.2 Inductive and Deductive Reasoning

6. What conclusion can you make about the sum of any two even integers?

$2+4=6$

$-4+-8=-12$

$-2+0=-2$

conclusion: The sum of any two even integers is even.

7. What conclusion can you make about the difference of any two odd integers?

$9-5=4$

$-1-5=-6$

$3-7=-4$

conclusion: The difference of any two odd integers is even.

8. What conclusion can you make about the product of an even and an odd integer?

$-2\cdot 3=-6$

$4\cdot 7=28$

$-8\cdot -1=8$

conclusion: The product of an even and an odd integer is even.

9. Use the Law of Detachment to make a valid conclusion. If an angle is a right angle, then the angle measures 90°. $\angle B$ is a right angle.

$\boxed{\angle B \text{ measures } 90°.}$

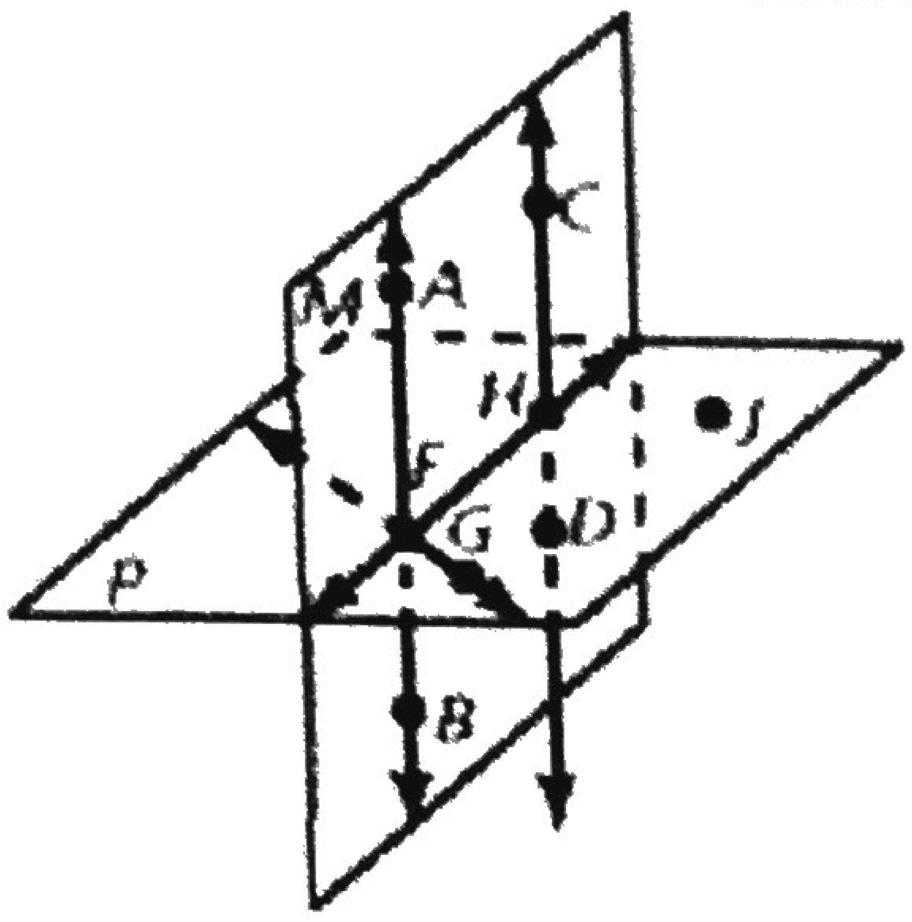
10. Use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements: If $x = 3$, then $2x = 6$. If $4x = 12$, then $x = 3$.

If $4x=12$, then $x=3$. If $x=3$, then $2x=6$.

$\boxed{\text{If } 4x=12 \text{, then } 2x=6.}$

2.3 Postulates and Diagrams

11. Use the diagram to make three statements that can be concluded and three statements that *cannot* be concluded. Justify your answers.



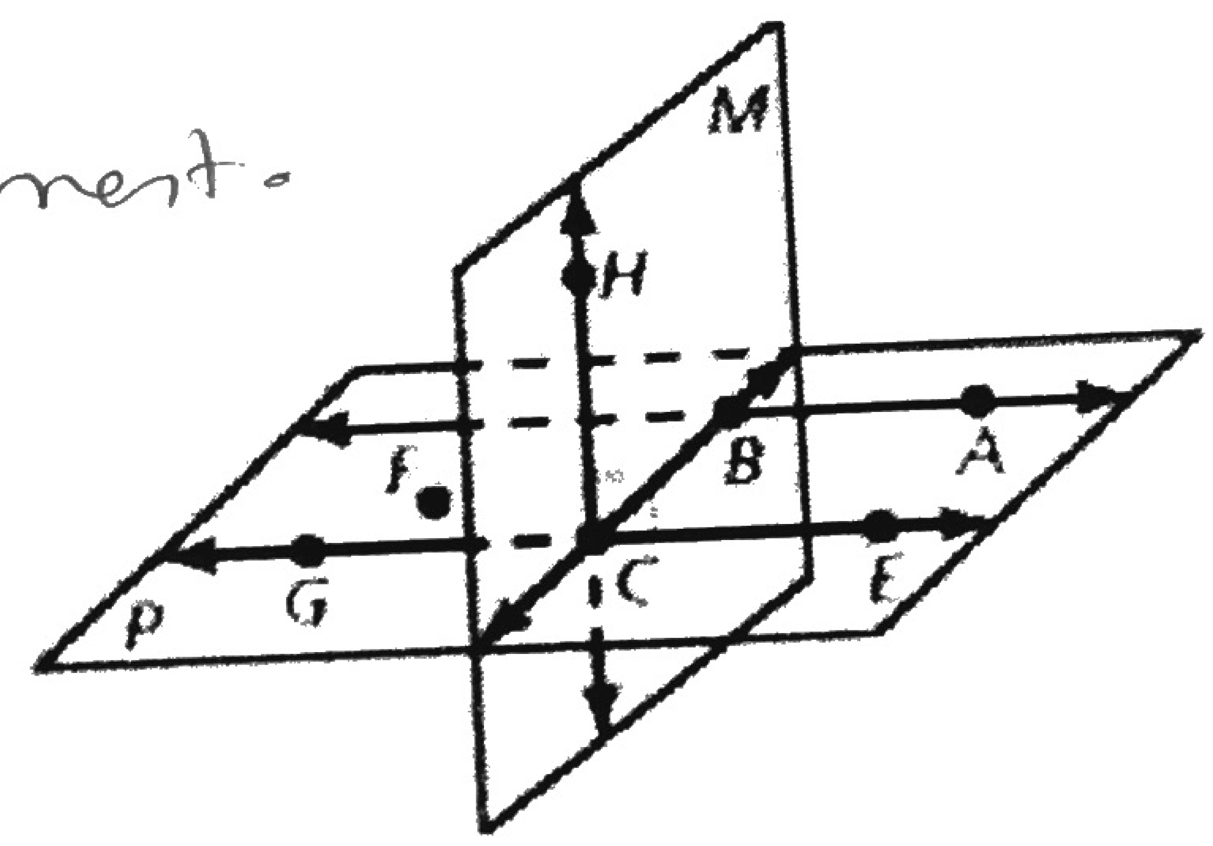
Can conclude

- Collinear points
- coplanar points
- linear pair / supp. ∠'s
- intersections
- vertical ∠'s

Cannot conclude

- ⊥ lines / right ∠'s
- ∥ (parallel) lines
- ≅

Use the diagram at the right to determine whether you can assume the statement.



12. Points A, B, C, and E are coplanar.

Yes, you can assume the statement.

13. $\overleftrightarrow{HC} \perp \overleftrightarrow{GE}$

Yes, you can assume the statement.

14. Points F, B, and G are collinear.

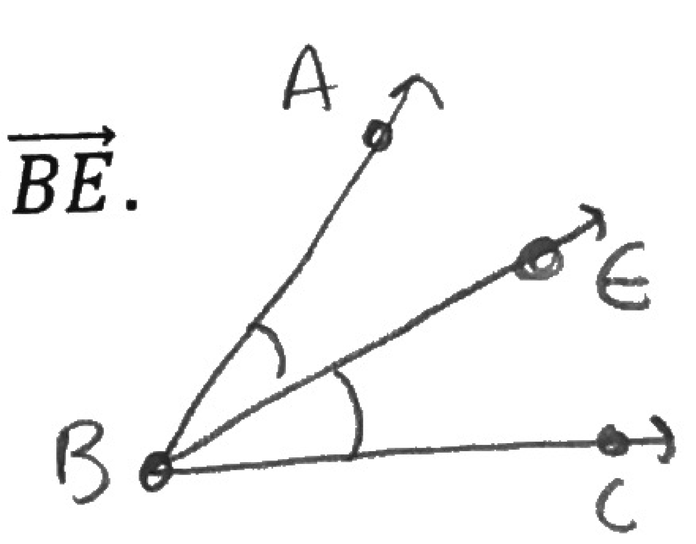
No, you cannot assume the statement (not on same line

15. $\overleftrightarrow{AB} \parallel \overleftrightarrow{GE}$

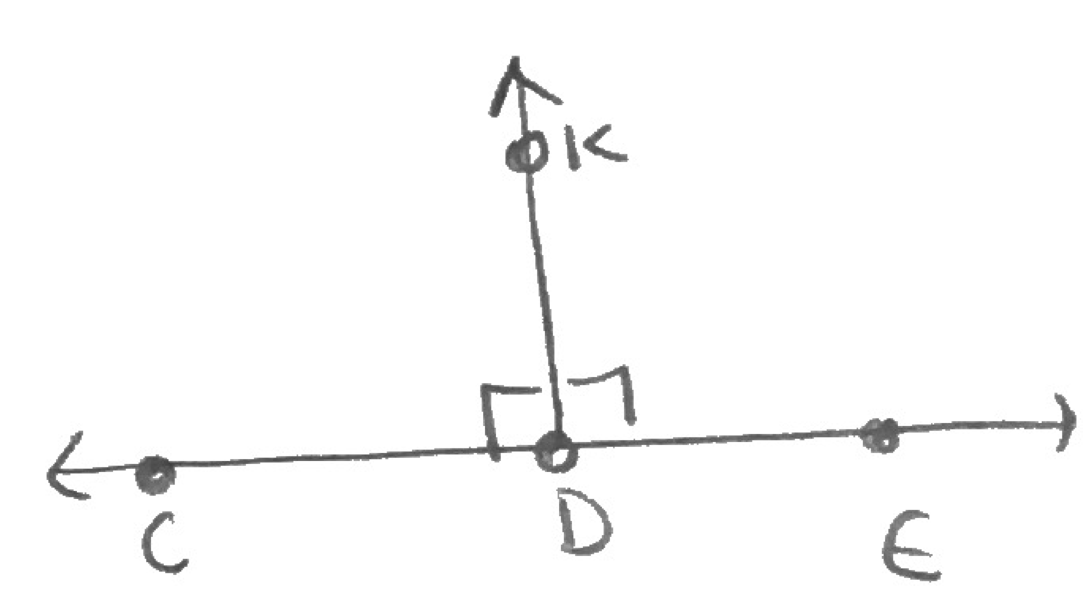
No, you cannot assume the statement (not marked)

Sketch a diagram of the description.

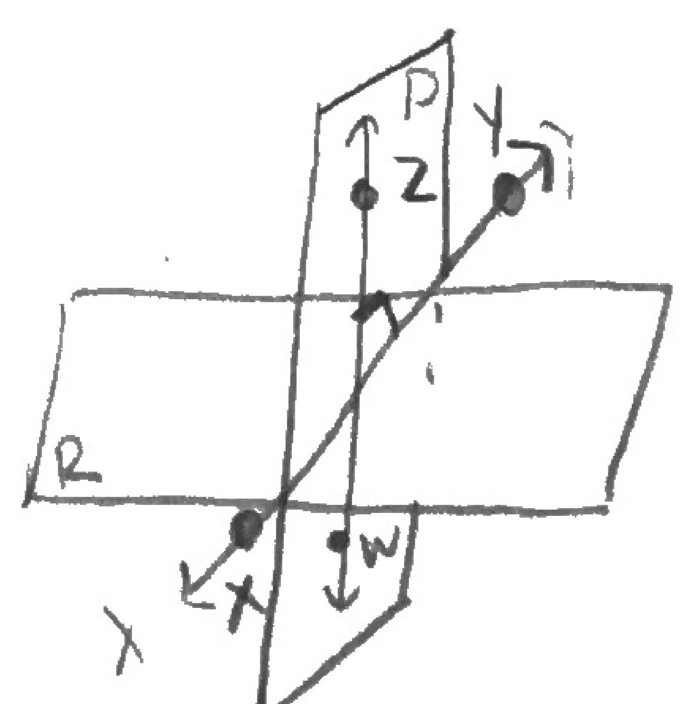
16. $\angle ABC$, an acute angle, is bisected by $\overrightarrow{BE}$.



17. $\angle CDE$, a straight angle, is bisected by $\overleftrightarrow{DK}$.



18. Plane P and plane R intersect perpendicularly in $\overleftrightarrow{XY}$. $\overline{ZW}$ lies in plane P.



2.4 Algebraic Reasoning

19. Solve $2(2x + 9) = -10$. Justify each step.

Equation	Reason
① $2(2x+9)=-10$	① Given
② $4x+18=-10$	② Distributive Property
③ $4x=-28$	③ Subtraction Property of =
④ $x=-7$	④ Division Property of =

Solve the equation. Justify each step.

20. $-9x - 21 = -20x - 87$

Equation	Reason
① $-9x-21=-20x-87$	① Given
② $11x=-66$	② Addition Prop. of =
③ $x=-6$	③ Division Prop. of =

21. $15x + 22 = 7x + 62$

Equation	Reason
① $15x+22=7x+62$	① given
② $8x=40$	② Subtraction Prop of =
③ $x=5$	③ Division Prop. of =

22. $3(2x + 9) = 30$

Equation	Reason
① $3(2x+9)=30$	① Given
② $6x+27=30$	② Distributive Property
③ $6x=3$	③ Subtraction Prop. of =
④ $x=\frac{1}{2}$	④ Division Prop. of =

23. $5x + 2(2x - 23) = -154$

Equation	Reason
① $5x+2(2x-23)=-154$	① Given
② $5x+4x-46=-154$	② Distributive Property
③ $9x-46=-154$	③ Combine like terms.
④ $9x=-108$	④ Addition prop. of =
⑥ $x=-12$	⑥ Division Prop. of =

Name the property of equality that the statement illustrates.

24. If $LM = RS$ and $RS = 25$, then $LM = 25$.

Transitive Property of Equality

25. $AM = AM$

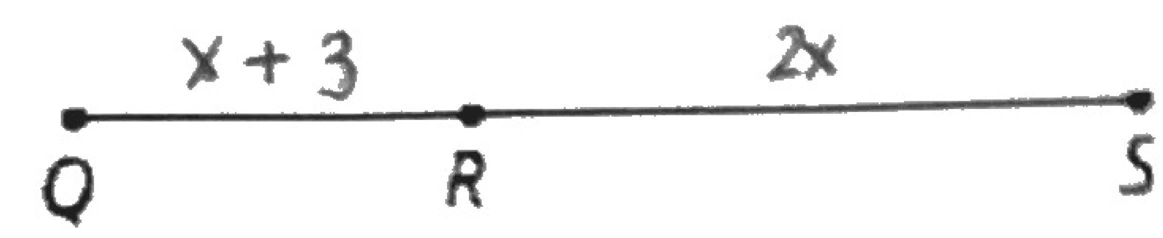
Reflexive Property of Equality

2.5 Proving Statements about Segments and Angles

26. Fill in the reason that justifies each step.

Given: $QS = 42$

Prove: $x = 13$



Statements	Reasons
1. $QS = 42$	1. Given
2. $QR + RS = QS$	2. Segment Addition Postulate
3. $(x + 3) + 2x = 42$	3. Substitution Property of =
4. $3x + 3 = 42$	4. Combine like terms.
5. $3x = 39$	5. Subtraction Property of =
6. $x = 13$	6. Division Property of =

Name the property that the statement illustrates.

27. If $\angle DEF \cong \angle JKL$, then $\angle JKL \cong \angle DEF$.

Transitive Property of ≅

28. $\angle C \cong \angle C$

Reflexive Property of ≅

29. If $MN = PQ$ and $PQ = RS$, then $MN = RS$.

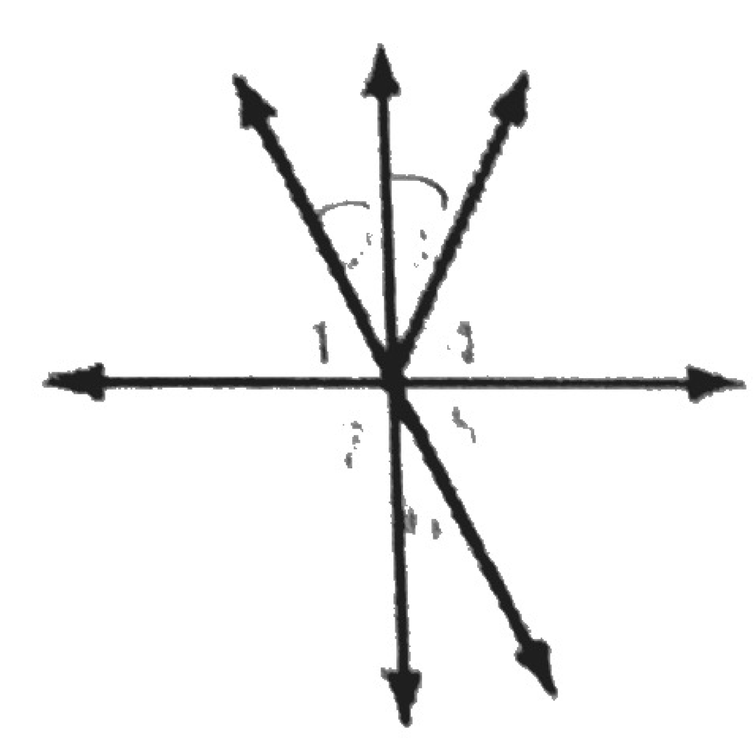
Transitive Property of =

2.6 Proving Geometric Relationships

31. Rewrite the two-column proof into a paragraph proof.

Given: $\angle 2 \cong \angle 3$

Prove: $\angle 3 \cong \angle 6$



Two-Column Proof

Statements	Reasons
1. $\angle 2 \cong \angle 3$	**1.** Given
2. $\angle 2 \cong \angle 6$	**2.** Vertical Angles Congruence Theorem
3. $\angle 3 \cong \angle 6$	**3.** Transitive Property of Angle Congruence

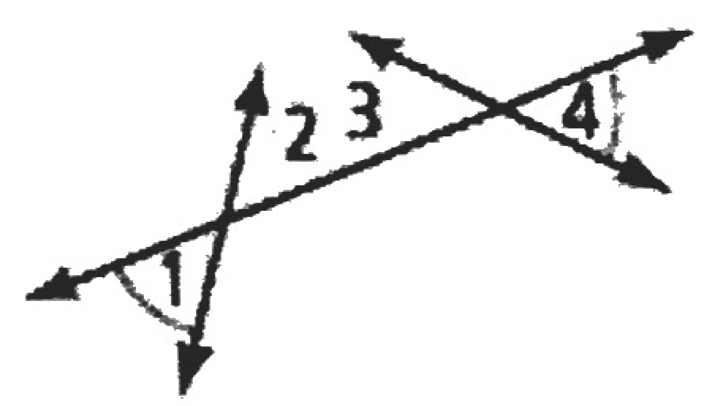
∠2 and ∠3 are congruent because it is given. By the Vertical Angles Congruence Theorem, ∠2 ≅ ∠6. By the Transitive Property of Angle Congruence, ∠3 ≅ ∠6

32. Write a proof.
Given: $\angle 3$ and $\angle 2$ are complementary. $m\angle 1 + m\angle 2 = 90°$
Prove: $\angle 3 \cong \angle 1$

$\angle 3$ and $\angle 2$ are complementary → $m\angle 3 + m\angle 2 = 90°$
Given
Def. of complementary $\angle$'s

$m\angle 1 + m\angle 2 = 90°$ → $m\angle 3 + m\angle 2 = m\angle 1 + m\angle 2$ → $m\angle 3 = m\angle 1$ → $\angle 3 \cong \angle 1$
Given
Transitive Prop. of =
Subtraction Prop. of =
Def. of $\cong$

33. Write a proof.
Given: $\angle 1 \cong \angle 4$
Prove: $\angle 2 \cong \angle 3$



$\angle 1 \cong \angle 4$
Given

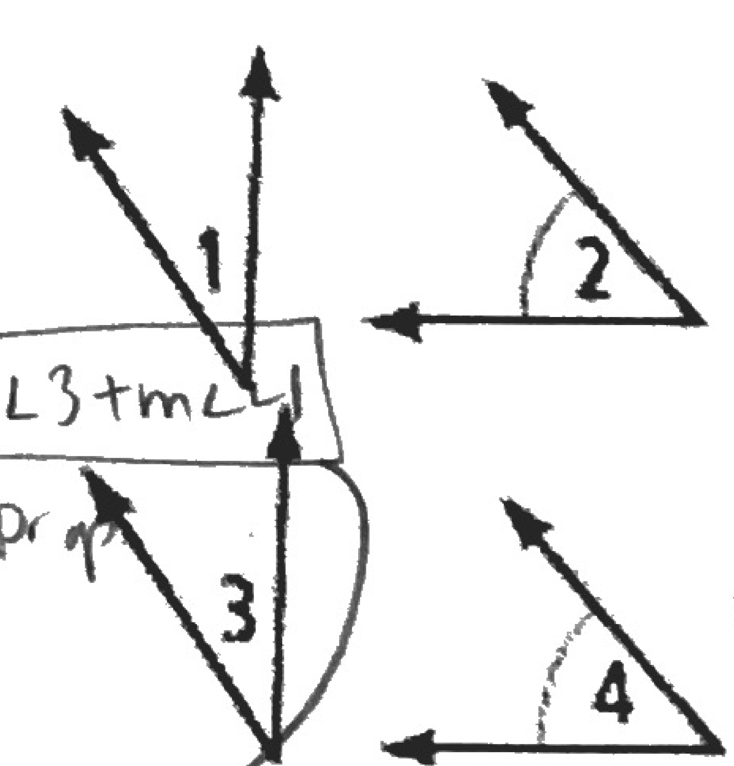
$\angle 2 \cong \angle 4$
Transitive Prop. of $\cong$

$\angle 1 \cong \angle 2$
Vertical $\angle$'s $\cong$ Theorem

$\angle 2 \cong \angle 3$
Transitive Prop. of $\cong$

$\angle 3 \cong \angle 4$
Vertical $\angle$'s $\cong$ Theorem

34. Write a proof.
Given: $\angle 1$ and $\angle 2$ are complementary. $\angle 3$ and $\angle 4$ are complementary. $\angle 2 \cong \angle 4$.
Prove: $\angle 1 \cong \angle 3$



$\angle 1$ and $\angle 2$ are complementary → $m\angle 1 + m\angle 2 = 90°$
Given
Def. of complementary $\angle$'s

$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$
Transitive Prop. of =

$\angle 3$ and $\angle 4$ are complementary → $m\angle 3 + m\angle 4 = 90°$
Given
Def. of complementary $\angle$'s

$\angle 1 \cong \angle 3$
Def. of $\cong$

$\angle 2 \cong \angle 4$ → $m\angle 2 = m\angle 4$ → $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$ → $m\angle 1 = m\angle 3$
Given
Def. of $\cong$
Substitution Prop. of =
Subtraction Prop. of